A Monte Carlo simulation method for system reliability analysis

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Abstract: Bases of Monte Carlo simulation are briefly described. Details of the application of Excel software to Monte Carlo simulation are shown with an analysis example. Three-component system is taken up and analysis is performed with the consideration of repair actions. Finally, it is shown that loop structure can be solved by Monte Carlo simulation method, which is realized by Excel software. The simulation results are compared with the analytical calculation results and good agreement is confirmed.

Keyword: Monte Carlo simulation; reliability analysis; repair of component; loop structure

1 Introduction

In this lecture note, explanations are given for a Monte Carlo simulation method for system reliability analysis.

Many system reliability analysis methods are proposed and used for probabilistic safety assessments (PSAs), mainly to assess the safety of nuclear power plants [1]. Although event tree and fault tree analyses are widely used in nuclear field, the system structures of nuclear plants are mostly very complex. System operational modes such as "phased mission" are sometimes complicated, and there are various dependencies between operation of subsystems or component failures. Components have many kinds of failure modes and failure distributions, and failed components are repaired and maintained in the actual operational conditions.

Taking into account these conditions, system reliability analysis through analytical models will become very difficult and restrictive. Simulation methods can straightforwardly treat these many complicated conditions in one analysis framework.

We first explain the bases of Monte Carlo simulation. Detailed explanations are given for the application of Excel software to Monte Carlo simulation. Three components system is taken up and analysis is performed with the consideration of repair actions. Finally, the Monte Carlo simulation method is applied to a system in which there is mutually supportive relation (loop structure). Reliability of a system with loop structure was not generally obtained in terms of the arithmetic operators of Boolean algebra, but it is shown that loop structure can be straightforwardly solved by this simulation method.

If the readers find out an advantage of this simulation method, kindly utilize Monte Carlo simulation to your analysis.

2 Monte Carlo simulation

The idea of Monte Carlo methods[2] is the generation of random events in a computer model, and this generation is repeated many times and count the occurrence number of a specific condition. Monte Carlo methods are often used in simulations of physical and mathematical systems.

During the wartime period, the first electronic computer ENIAC was developed at the University of Pennsylvania. It was used for the calculation of thermonuclear problem in Manhattan Project. After the war, Stan Ulam, who was well versed with statistical sampling techniques, had an idea to use ENIAC's miraculous ability for this technique. He discussed with John von Neumann. In 1947, Neumann showed a detailed outline of a possible

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statistical approach to solving the problem of neutron diffusion in fissionable material. Additionally, Neumann conceived the algorithm for generating uniformly distributed pseudo-random numbers. This was the start of the Monte Carlo method[3]. The method was named after the Monte Carlo Casino, a famous casino where many people, including Ulam's uncle, would often gamble away their money.

The simulation is treated as a series of real experiments, and statistical inference will be used to estimate the confidence intervals for the performance. As an example, consider component failure phenomenon. Start with a sound state of a component and observe it for certain time duration *t*. The component will be in sound or failed state at time *t* with the aid of random events in a computer model. Repeat this observation many times and collect the events the component is in sound state. Fraction of the number of sound states over total observation number gives the success probability of a component at time *t*, and this probability is the reliability *R(t)* of this component. Numerical value of success probability is easily obtained, and the success probability reflects an assigned failure model, which is used in the generation of random events.

The Monte Carlo simulation allows us to consider various aspects of system characteristics which cannot be easily captured by analytical methods such as K-out-of-N success criteria, redundancies, phased mission, stand-by condition, aging effects, repair and maintenance for components. We can avoid the restrictive modeling assumptions that had to be introduced to fit the models to available solutions. The Monte Carlo simulation is used to complement these theoretical derivations.

The basic procedure of Monte Carlo method is: (1) Define a domain of possible events, (2) Generate events randomly, (3) Perform deterministic judgments of system states based on the events, (4) Count the occurrence number of a specific system state among total observations.

A weakness of the Monte Carlo method is the computing time expended particularly when we deal with a large complex system.

3 Monte Carlo simulation by Excel

Let us first consider the failure process of one component with constant failure rates (standby: λ_S, operating: λ_O) and demand probability Q_D. The reliability *R(t)* in standby state is governed by the following equation:

$$\frac{dR(t)}{dt} = -\lambda_S R(t). \quad (1)$$

With the initial condition *R(t=0)=1.0*, reliability becomes:

$$R(t) = \exp\{-\lambda_S t\}. \quad (2)$$

Suppose that a demand for the start of operation is given at time t_1, then the reliability is:

$$R(t_1) = Q_D \cdot \exp\{-\lambda_S t_1\}. \quad (3)$$

After the start of operation, the reliability is expressed by the following equation:

$$R(t) = Q_D \cdot \exp\{-\lambda_S t_1\} \cdot \exp\{-\lambda_O (t - t_1)\}. \quad (4)$$

Let us denote the reliability of a component at a certain time t_2 as $R(t_2)$. After a small time duration Δt from t_2, the reliability becomes:

$$R(t_2 + \Delta t) = R(t_2) \exp\{-\lambda_{S,O} \cdot \Delta t\}. \quad (5)$$

The above relations are schematically illustrated as shown in Fig. 1.

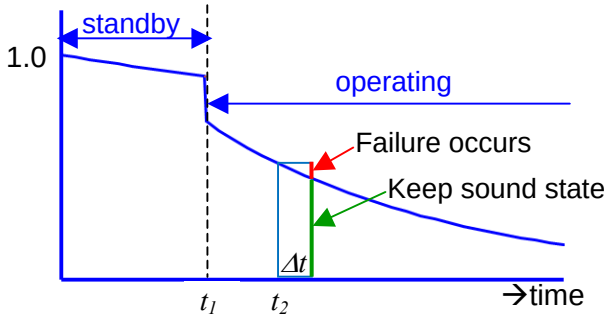

Fig. 1 Reliability of a component with time.

Failure of a component is a stochastic phenomenon. For a specific observation, failure happens at a certain time, for example t_f. Before time t_f, the component is completely in sound condition and after time t_f the component loses its function, that is, the

component is in failed state with the probability of 1.0. If we make numerous amount of observations for the same component under the same condition (as idealistic experiments), the distribution of t_f is obtained, and also the number of the cases in which component is in sound state at specific time t. The proportion of this number against total observation number becomes the reliability $R(t)$.

The ratio $R(t_2+\Delta t)/R(t_2)$ equals $\exp\{-\lambda_{S,O}\cdot\Delta t\}$ as seen from Eq.(5). This is the probability that the component is still in sound state after Δt time passed. Transition from sound state to failed state during time interval Δt can be found by using random number between 0 to 1. If random number is greater than $\exp\{-\lambda_{S,O}\cdot\Delta t\}$, it is judged that the component failed during this time interval Δt in a specific observation.

If failure rates are not constant values but a function of time, then Eq. (5) becomes:

$$R(t_2+\Delta t)=R(t_2)\exp\{-\int_{t_2}^{t_2+\Delta t}\lambda(t)dt\}. \qquad (6)$$

If analytical integration is difficult, the following approximation is useful:

$$\int_{t_2}^{t_2+\Delta t}\lambda(t)dt\approx\lambda(t_2)\cdot\Delta t \qquad (7)$$

The above transition process can be simulated by the Excel software as shown in Fig. 2. In this case, $\lambda_S=0.01/\text{min.}$, $\lambda_O=0.03/\text{min}$, $Q_D=0.9$ are assumed.

	A	B	C	D	E	F
1	Time	Standby or Operartion	Random number	Success or Failure	Cumulative state	Counting
2	0	S(0)	0.166570291	1	**1**	
3	2	S(0-2)	0.388347194	1	**1**	
4	4	S(2-4)	0.628607714	1	**1**	
5	6	S(4-6)	0.792436156	1	**1**	
6	8	S(6-8)	0.020459856	1	**1**	
7	10	S(8-10)	0.536262478	1	**1**	1
8	10	Q_D	0.417410082	1	**1**	1
9	12	O(10-12)	0.526036816	1	**1**	
10	14	O(12-14)	0.260857271	1	**1**	
11	16	O(14-16)	0.99626777	0	**0**	
12	18	O(16-18)	0.555622379	1	**0**	
13	20	O(18-20)	0.796045007	1	**0**	0
14	22	O(20-22)	0.044911903	1	**0**	
15	24	O(22-24)	0.986462011	0	**0**	
16	26	O(24-26)	0.789939883	1	**0**	
17	28	O(26-28)	0.909475007	1	**0**	
18	30	O(28-30)	0.35902208	1	**0**	0

Fig.2 Simulation by Excel table for one component.

Time span of 30 minutes are divided into 15 sections by each 2-minute time duration, and they are written in column A. Component in standby state is denoted as S(0-2), and one at operating state as O(10-12) in column B. Demand for the start of operation (Q_D) is given at 10 minutes. Uniformly distributed random number is generated in interval ［0,1］ by Excel function RAND( ). Random numbers are given at each time section, in column C.

In column D, transition from sound state to failed state is judged. In line 3, a random number of 0.388347194 is given. This value is smaller than the value of 0.9801987, which is calculated by $\exp(-\lambda_S\Delta t)=\exp(-0.01\times 2\text{min})$. In this case, we judge the component maintains a sound state during 0 to 2 minutes. This judgment is expressed by "1" in the cell D3 in Excel table.

In cell C8, a random value of 0.417410082 is given, which is smaller than the value of 0.9 (= demand probability Q_D). It means that the component successfully starts the operation. This is denoted by "1" in the cell D8.

In cell C11, a random value of 0.99626777 is given, and it is larger than the value of 0.941765, which is calculated by $\exp(-\lambda_O\Delta t)=\exp(-0.03\times 2\text{min})$. It means that the component unfortunately fails during 14 to 16 minutes. This judgment is expressed by "0" in the cell D11.

In column E, state of the component is expressed at each time. Sound state is denoted by "1", and failed state by "0". In this model, repair is not considered. Therefore, once failure occurs, component is thereafter maintained at failed state. This means that judgments in D12 to D18 are not used in this case.

Figure 2 shows the results of one experiment, that is, one case that happened by chance. Now, collect the numbers of success cases for the time 10 minute, just after the demand, 20 minute and 30 minute, by counting the number of "1" in the cells F7, F8, F13 and F18 in a series of numerous experiments (trials). Figure 3 shows the results.

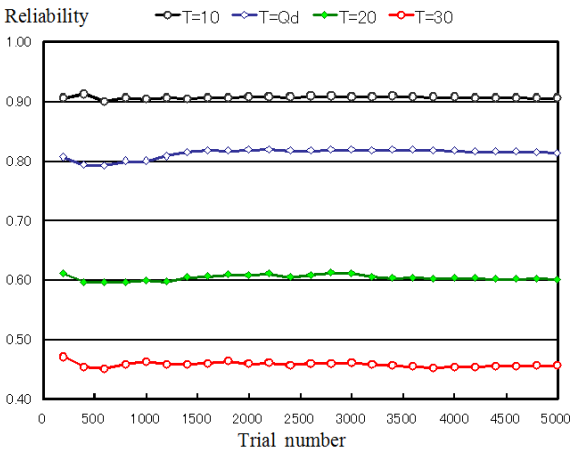

Fig.3 Simulation results for one component.

For the reliability of one component with constant failure rates, analytical calculation can be easily performed and results are as follows:

$$R(t=10)=\exp\{-0.01\times 10\}=0.905. \quad (8)$$

$$R(10;\ after\ Demand)=Q_D\times 0.905=0.814. \quad (9)$$

$$R(20)=R(10)\cdot\exp\{-0.03\times 10\}=0.603. \quad (10)$$

$$R(30)=R(10)\cdot\exp\{-0.03\times 20\}=0.447. \quad (11)$$

The simulation results shown in Fig. 3 give good agreement at 5000 trials with the above analytical calculation.

4 Three-component system with repair action

Now, take an example of a three-component system and monitor the operating state of the system with the consideration of repair action. Let us denote the probability of operating state of a component at time t as $P_O(t)$. If failed component is continuously repaired with constant repair rate μ, the probability $P_O(t)$ is governed by the following equation:

$$\frac{dP_O(t)}{dt}=-\lambda P_O(t)+\mu\{1-P_O(t)\} \quad (12)$$

With the initial condition $P_O(t=0)=P(0)$, $P_O(t)$ becomes as follows:

$$P_O(t)=\frac{\mu}{\mu+\lambda}-\frac{\mu-(\lambda+\mu)P(0)}{\mu+\lambda}\exp(-(\lambda+\mu)t). \quad (13)$$

Operating probability of one component can be obtained by an Excel simulation as shown in Fig. 4.

In this case, the following assumptions are made: $\lambda_S=0.01/min.$, $\lambda_O=0.03/min.$, $Q_{DA}=0.9$, and $\mu=0.03/min.$

"Random number(1)" in column C is used for the judgment of failure and "Random number(2)" in column E is used for repair. If random number(2) is greater than $\exp\{-\mu\cdot\Delta t\}$, it is judged that the component is repaired during this time interval Δt in a specific observation. The success of repair action is expressed by "1" in the column F.

The state of component (column G) is judged as follows. If the component is in sound state in previous time interval and failure occurs ("0" in column D), component state is changed to failed state ("0" in column G). If the component is in failed state in previous time interval and repaired ("1" in column F), component state is changed to sound state ("1" in column G). In other conditions, component state is unchanged.

	A	B	C	D	E	F	G	H
1	Time	Operation mode	Random number(1)	Success or Failure	Random number(2)	Repair	State of Component	Operation
2	0	S(0)	0.257595	1	0.843243	1	1	0
3	2	S(0-2)	0.988118	0	0.606122	0	0	0
4	4	S(2-4)	0.864059	1	0.961135	1	1	0
5	6	S(4-6)	0.678167	1	0.817631	0	1	0
6	8	S(6-8)	0.688674	1	0.748942	0	1	0
7	10	S(8-10)	0.818004	1	0.224544	0	1	0
8	10	Q_{DA}	0.15712	1	0.154052		1	1
9	12	O(10-12)	0.916451	1	0.987256	1	1	1
10	14	O(12-14)	0.951243	0	0.435951	0	0	0
11	16	O(14-16)	0.254775	1	0.807802	0	0	0
12	18	O(16-18)	0.275766	1	0.364012	0	0	0
13	20	O(18-20)	0.019932	1	0.615093	0	0	0
14	22	O(20-22)	0.651189	1	0.995636	1	1	1
15	24	O(22-24)	0.355673	1	0.033883	0	1	1
16	26	O(24-26)	0.861875	1	0.594138	0	1	1
17	28	O(26-28)	0.020261	1	0.581893	0	1	1
18	30	O(28-30)	0.271774	1	0.636244	0	1	1
19	32	O(30-32)	0.391741	1	0.810189	0	1	1
20	34	O(32-34)	0.902077	1	0.501718	0	1	1
21	36	O(34-36)	0.315774	1	0.660723	0	1	1
22	38	O(36-38)	0.611354	1	0.886348	0	1	1
23	40	O(38-40)	0.749063	1	0.67272	0	1	1

Fig.4 Simulation for one component with repair action.

In Fig. 4, failure occurs during time interval 0 to 2 minutes ("0" in cell D3), then component state is changed to failed state ("0" in cell G3). During 2 to 4 minutes repair action has been succeeded ("1" in cell F4), then component state is changed to sound state ("1" in cell G4). Component is successfully started ("1" in cell D8), and the operation started ("1" in cell H8). During the standby state, component is not operated ("0" in cells H2 to H7). Failure again occurs ("0" in cell D10), consequently rendering component

to be in failed state ("0" in cell G10) and operation stops ("0" in cell H10).

Success judgment appears in cell D11. However in this case, component is already in failed state and this judgment is not used. Fortunately repair is made ("1" in cell F14) and component is in sound state, and is placed in operating condition. Figure 5 shows the simulation results.

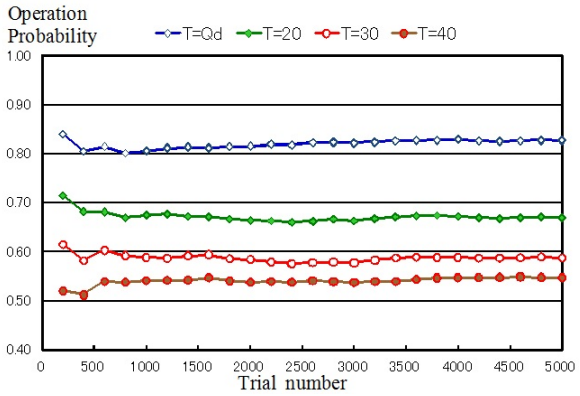

Fig.5 Simulation results for one component with repair action.

Analytical values can be obtained by Eq. (13), as follows:

$$P_O(t=10,\ after\ Demand) = Q_{DA}\exp\{-\lambda_S \times 10\}$$
$$= 0.9\cdot\exp\{-0.01\times 10\} = 0.825. \quad (14)$$

$$P_O(20) = \frac{\mu}{\mu+\lambda_O} - \frac{\mu-(\lambda_O+\mu)P_O(10)}{\mu+\lambda_O}\cdot\exp\{-(\mu+\lambda_O)\times 10\}$$
$$= 0.5 - \frac{0.03-0.06\times 0.825}{0.06}\exp\{-0.6\} = 0.678. \quad (15)$$

$$P_O(30) = 0.598. \quad (16)$$

$$P_O(40) = 0.554. \quad (17)$$

The simulation results at 5000 trials again give good agreement with the analytical values. Compared to the results shown in Fig.3, the repair effects are apparent.

Now, consider the three-component system as shown in Fig. 6.

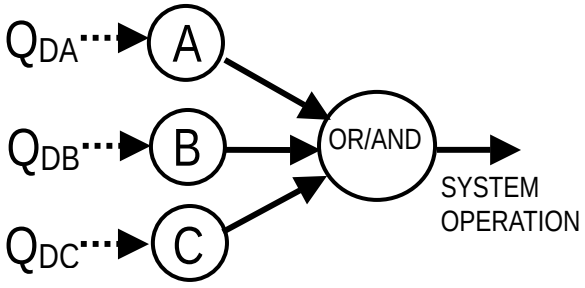

Fig.6 Three-component system.

Failure data (λ_S, λ_O, Q_D, μ) for components A, B and C are assumed as (*0.01/min., 0.03/min., 0.9, 0.03/min.)*, (*0.01/min., 0.05/min., 0.9, 0.01/min.)* and (*0.006/min., 0.01/min., 0.99, 0.03/min.)*, respectively. Commencement of operation for component A, B and C are given at 10, 20 and 30 minutes, respectively.

For the simulation analysis, first prepare the Excel table for each component as shown in Fig. 4. Then, express system operational logic in Excel table. For example, "OR" logic between three components is written as "IF(I23+I74+I100>=1,1,0)" in a cell (J23). Here, cells I23, I74 and I100 have values "1" when components A, B and C are in operating state at time 40 minutes, respectively. Finally count appeared number of "1" in cell J23. The ratio of this number to the total observation number becomes the probability of the system operation by "OR" logic at the 40 minutes.

Any kind of system operational logic can be written by logical formula. Therefore, system operation or configuration can be easily modeled in this simulation method. Figure 7 shows the simulation result for "OR" logic.

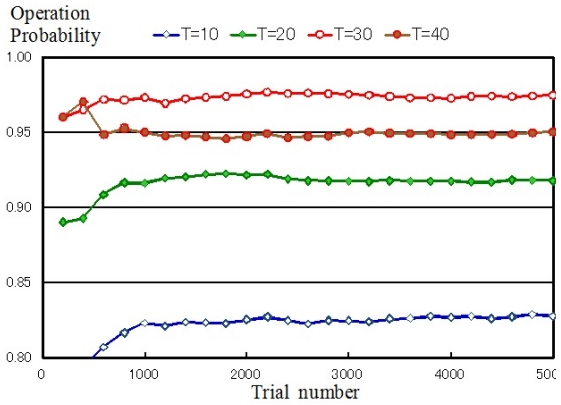

Fig.7 Simulation results for three component system.

5 Analysis of loop structure by Monte Carlo simulation

For a system, which has logical loop structure(s), the Boolean relation has to be described with unknown variable(s). If we try to solve the Boolean equation(s) with unknown variable(s), we encounter infinite circulation of the unknown variable(s). Logical loop was not generally solved in terms of the arithmetic operators of Boolean algebra[4-7]. The authors have shown that loop structure could be solved by analytical procedure and also by a simulation method[8]. In this lecture note, detailed explanations regarding the procedure to solve a loop structure by Monte Carlo simulation method are given.

As a fundamental configuration of a loop structure, we think of a system as shown in Fig. 8. Components S1 and S2 are self sustained type (SS-type) components, and components A, B and C are G-type components, which require support for their operation. This is an additional condition in the analysis of loop structure, not required in previous cases.

Time sequence of the starts of operation is set as t_{s1} (= 4 min.), t_{s2}(=8 min.), t_B(=12 min.), t_A(=16 min.) and t_C(=20 min.).

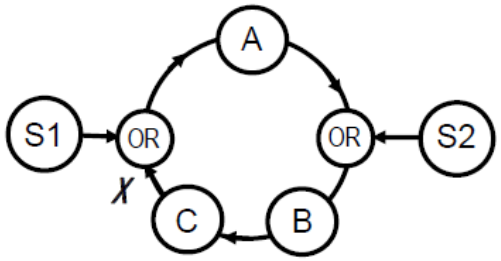

Fig.8 Loop structure for simulation analysis.

The operating probability of component C has been obtained by the following analytical equation [9]:

$$X(t)= S1(\tau_C)\, S2(\tau_A)\cdot A(t)B(t)C(t)+ S2(t)B(t)C(t) \qquad (18)$$

In Eq. (18) both τ_A(t=16 min.) and τ_C(t=20 min.) are not variables but express fixed time points. After the establishment of loop operation, components S1 and S2 are stopped at times of 24 min. and 28 min. Without the supports of SS-type components, the loop structure continues its operation and the probability is:

$$X(t)= S1(\tau_C)\, S2(\tau_A)\cdot A(t)B(t)C(t). \qquad (19)$$

Components' states of S1 and A are obtained by Excel tables as shown in Figs.9 and 10, respectively. Failure data (λ_S, λ_O, Q_D) are assumed for components S1, S2, A, B and C as (*0.001/min., 0.03/min., 0.9)*, (*0.001/min., 0.03/min., 0.9)*, (*0.003/min., 0.01/min., 0.95)* , (*0.003/min., 0.01/min., 0.95)* and (*0.003/min., 0.01/min., 0.95)*, respectively. For the simplicity, repair action is not considered.

Time	Operation mode	Random number	Success or Failure	State of Component	Operating state
0	S(0)	0.000603	1	1	0
2	S(0-2)	0.650115	1	1	0
4	S(2-4)	0.479275	1	1	0
4	Qd-S1	0.516484	1	1	1
6	O(4-6)	0.18981	1	1	1
8	O(6-8)	0.771356	1	1	1
10	O(8-10)	0.301401	1	1	1
12	O(10-12)	0.845381	1	1	1
14	O(12-14)	0.658205	1	1	1
16	O(14-16)	0.392672	1	1	1
18	O(16-18)	0.770618	1	1	1
20	O(18-20)	0.703634	1	1	1
22	O(20-22)	0.166524	1	1	1
24	O(22-24)	0.71371	1	1	1
24	Qd-S1X	–	–	–	0
26	–	–	–	–	0
28	–	–	–	–	0
30	–	–	–	–	0
32	–	–	–	–	0
34	–	–	–	–	0
36	–	–	–	–	0
38	–	–	–	–	0
40	–	–	–	–	0

Fig.9 Excel table for SS-type component S1.

Time	Operation mode	Random number	Success or Failure	State of Component	Operable state
0	S(0)	0.203622	1	1	0
2	S(0-2)	0.038305	1	1	0
4	S(2-4)	0.553231	1	1	0
6	S(4-6)	0.892056	1	1	0
8	S(6-8)	0.641034	1	1	0
10	S(8-10)	0.751363	1	1	0
12	S(10-12)	0.378489	1	1	0
14	S(12-14)	0.401957	1	1	0
16	S(14-16)	0.96376	1	1	0
16	Qd-A	0.826852	1	1	1
18	O(16-18)	0.35546	1	1	1
20	O(18-20)	0.428757	1	1	1
22	O(20-22)	0.202294	1	1	1
24	O(22-24)	0.543767	1	1	1
26	O(24-26)	0.740703	1	1	1
28	O(26-28)	0.503788	1	1	1
30	O(28-30)	0.690876	1	1	1
32	O(30-32)	0.988171	0	0	0
34	O(32-34)	0.209107	1	0	0
36	O(34-36)	0.579538	1	0	0
38	O(36-38)	0.007939	1	0	0
40	O(38-40)	0.979722	1	0	0

Fig.10 Excel table for G-type component A.

Figure 9 is a table for component S1. During the standby period, operating state is "0" (not operating). After the successful start of operation, it becomes operating state "1", because it is a SS-type component. S1 is stopped at time of 24 min., then operating states are "0" after the time of 24 min.

Figure 10 shows a table for G-type component A. This type of component requires the support for its operation. Therefore, last column is "operable state" and "1" in this column means that a component is in sound state and has a possibility to be operated with the proper support. In this case, failure occurs during a time interval from 30 to 32 min. and component A has no ability to be operated after 32 min.

System with a loop structure (shown in Fig. 8) is simulated by an Excel table as shown in Fig. 11. Component states are judged for each time or time interval. At the right side of the component name, operating state is shown as "1" (operating) or "0" (not operating).

At a time of 12 min., components S1 and S2 are already in operating state, and components A and C are in standby condition. Demand for start of operation is given to component B, and operation is judged by the following logical formula:

$$\text{IF}(N80*(G124+I125)>=1,1,0) \qquad (20)$$

Successful start ("1") or not ("0") is taken from the cell N80 in the component B's table like Fig. 10. Cell G124 is next to component name A and gives A's operating state. Cell I125 is next to component name S2 and has a value of "1" (S2 is operating), in this case. Indeed, component B can be supported only by component S2 at the time 12 min.

Next box is prepared for time interval of 12 to 16 min., and operating states are evaluated for time of 16 min. Component B's state is judged by the following formula:

$$\text{IF}(N82*G126*(G129+I130)>=1,1,0) \qquad (21)$$

C	D	E	F	G	H	I
T=12(B starts)						
S1	1		**A**	0		
					S2	1
	C	0	**B**	1		
T=12-16						
S1	1		**A**	0		
					S2	1
	C	0	**B**	1		
T=16(A starts)						
S1	1		**A**	1		
					S2	1
	C	0	**B**	1		
T=16-18						
S1	1		**A**	1		
					S2	1
	C	0	**B**	1		
T=18-20						
S1	1		**A**	1		
					S2	1
	C	0	**B**	1		
T=20(C starts)						
S1	1		**A**	1		
					S2	1
	C	1	**B**	1		
T=20-22						
S1	1		**A**	1		
					S2	1
	C	1	**B**	1		
T=22-24						
S1	1		**A**	1		
					S2	1
	C	1	**B**	1		
T=24(S1 stops)						
S1	0		**A**	1		
					S2	1
	C	1	**B**	1		
T=24-26						
S1	0		**A**	1		
					S2	1
	C	1	**B**	1		

Fig.11 Excel table for a loop structured system.

Cell N82 gives operability of component B at time of 16 min. Cell G126 gives the operating state of B at the end of previous time interval (=12 min. in this case). The reason of this condition is as follows. If component stops operation due to failure or lack of support, it will not go back to operating state without additional demand of start. Therefore, continuous operation in the past is essential for the operation at a specific time. Soundness of a component and existence of support are not sufficient for the judgment of operating state of a component at specific time.

At time of 16 min., component A is started. Component A's operation after the start is judged by the following logical formula:

$$\text{IF(N57*(D134+E131)>=1,1,0)} \qquad (22)$$

Cell N57 is in the component A's table, and gives the information of successful start ("1") or not ("0"). Possible supports are from S1 (cell D134) and from C (cell E131). The system has loop structure and components C and A are connected. Thus, the support by C also has to be considered. In the present operational sequence, support by C becomes effective after the time of 20 min. (after the start of C).

In the time interval of 16 to 18 min., component A's operation is judged by the following logical formula:

$$\text{IF(N58*G134*(D139+E136)>=1,1,0)} \qquad (23)$$

Cell G134 gives the operating state of A at the previous time (=16 min. in this case). This is the same situation of G126 in Eq. (21).

In a similar way, operating states of the components are identified step by step with time. At time of 24 min., S1 is stopped, and operating state becomes "0" in the cell D164 (right side of component name "S1"). But the states of other components are operating ("1") with the support of S2 or mutual supports between components A, B and C.

The operating probability of component C is obtained by counting the number of "1" in the cells next to the component name "C" through the numerous number of observations. The results are shown in Fig. 12.

Observation is made for the time t=18 min. (Component C starts), t=24 min. (Component S1 stops), t=28 min. (Component S2 stops) and t=40 min.

Operating probabilities at time of 24 min. and just after the stop of S1 have exactly the same values.

This means that S1 has no contribution to the probability at time of 24 min. This is also seen in Eq. (18).

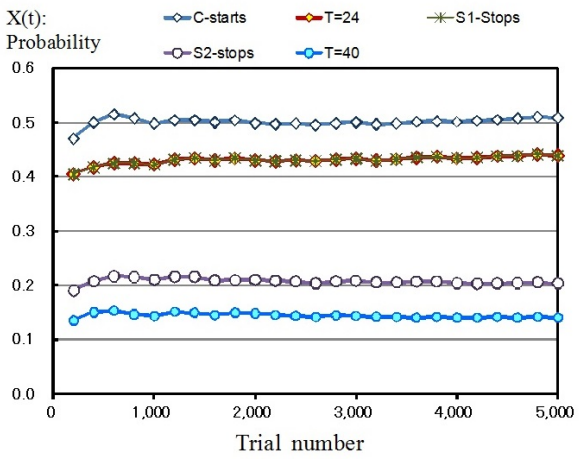

Fig.12 Results of simulation analysis for a loop structured system.

The values of $X(t)$ can be calculated from Eq. (18) or Eq.(19) as follows:

$$X(20,\ C\text{-}starts) = 0.5004. \qquad (24)$$
$$X(24) = 0.4349. \qquad (25)$$
$$X(28,\ S2\text{-}stops) = 0.2018. \qquad (26)$$
$$X(40) = 0.1498. \qquad (27)$$

For the calculation of $X(20)$, $X(24)$, the dependency between the first and second terms in Eq. (18) has been considered. The simulation results at 5000 trials give good agreement with the analytical values.

6 Conclusion

In this lecture note, explanations have been given for a Monte Carlo simulation method for system reliability analysis. Monte Carlo method can straightforwardly treat any kind of system structure and operating conditions. We therefore can avoid the restrictive modeling assumptions that had to be introduced in analytical methods.

The basic idea of Monte Carlo simulation has been described. Detailed explanations have been given for the application of Excel software to Monte Carlo simulation. Modeling method to treat repair actions has been explained in detail and three-component system has been analyzed. Finally, it has been shown that loop structure can be solved by this simulation method. The simulation results have been compared with the analytical calculation results and good agreement has been confirmed in all cases.

It is the author's hope that readers find out an advantage of the simulation method presented here, and utilize Monte Carlo simulation for futures research activities.

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