# Bayes' theorem and its application to nuclear power plant safety

**MATSUOKA Takeshi**[1, 2]

1. *Collage of Nuclear Science and Technology, Harbin Engineering University No.145-1, Nantong Street, Nangang District, Harbin, Heilongjiang Province, 150001 China*
2. *Center for Fundamental Education, Utsunomiya University, 350 Mine-machi, Utsunomiya City, 321-8505 Japan (mats@cc.utsunomiya-u.ac.jp)*

**Abstract:** Bayes' theorem has been paid in much attention for its application to Probabilistic Safety Assessment (PSA). In this lecture, the basis for understanding Bayes' theorem is first explained and how to interpret the Bayes' equation with respect to the pair of conjugate distributions between prior distribution and likelihood. Then for the application to PSA, component failure data are evaluated by Bayes' theorem by using the examples of demand probability of the start of diesel generator and failure of pressure sensor. Frequencies of nuclear power plant accidents are also evaluated by Bayes' theorem for the example case of frequency of "fires in reactor compartment" and "core melt" frequency with the experience of Fukushima dai-ichi accidents. Lastly, several contrasting arguments are introduced briefly between favorable and critical peoples regarding the Bayes' methods.

**Keyword:** Bayes' theorem; nuclear plant safety; expert opinion; component failure rate

## 1 Introduction

Methods of Probabilistic Safety Assessment (PSA) have been widely used for the evaluation of nuclear power plant safety[1]. It has been realized, however, that the methods of PSA have to deal with the rarity of events with lack of meaningful statistical data, where traditional statistical methods have not be well applicable to PSA. Therefore, Bayes' theorem has been made much attention for its application to PSA.

In this paper, we first discuss the interpretation of probability to highlight the contrasting views of "subjectivistic" and "frequentistic" probabilities. Then Bayes' theorem is explained in detail on how to interpret the Bayes' equation. Explanation is also given for the concept of the pair of conjugate distributions between prior distribution and likelihood. It is also shown that sequential evaluation by Bayes' method gives the same result with the lump evaluation.

Secondly, component failure data will be evaluated by the Bayes' theorem by using example of demand probability for the start of diesel generator and failure probability of pressure sensor. Also the frequency of nuclear power plant accidents is evaluated for the case of frequency of "fires in reactor compartment" and "core meltdown" frequency with the experience of Fukushima Dai-ichi accidents.

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However, there are many discussions between favorable and critical peoples to the Bayes' methods. Therefore, their contrasting opinions are briefly introduced in the last part of this lecture.

## 2 Probability, frequency and event

There are two major different interpretations of probability: probabilities of subjective and frequentistic viewpoints [2].

In subjective view, the probability corresponds to a "personal belief" (a degree of belief), for which rationality and consistency are required.

In frequentistic view, the probability can be obtained from the results of infinite experiment. In each trial, the "occurrence" or "no occurrence" of event is recorded. It is assumed that relative occurrence frequency converges to a certain limit value, and it becomes occurrence probability of the event. If we randomly chose a set of data from original total data, this limit value does not change. Idealized or true value of probability is deduced from experimental results. Any given experiment can be considered as one of possible repetitions of the same experiment, each capable of producing statistically independent results. Frequentist can use this probability as true if there are many events. In other word, probability cannot be assigned to a single event. New evidence (data) is only treated as sample information.

Therefore, frequentist method will become equivalent to subjective method when the evidence is very strong and it is treated as if a single event.

It is necessary to clarify the difference between probability and frequency. Frequency should be reduced from results of experiments to become measurable.

Usually, we use information other than observed facts for the establishment of belief. In subjective view, probability is a quantitative expression of state of knowledge. It cannot be obtained by experimental measurement.

An analyst has the probability which is reflected by his/her state of knowledge, and it satisfies the law of probability. The probabilities have a kind of values, and they are comparable with one of following three relations:

A=B, A＜B, A＞B.

Also transitive law can be applied as follows,

A＞B, B＞C → A＞C.

Event is described by a meaningful statement which can be determined “true” or “false”. Therefore, “core melt down occurs probably in the next 5 years” is not an event. Also “the probability of core melt down is $3x10^{-5}$/year” is not an event, because probabilistic description cannot be determined as “true” or “false”. “System A is safe”, “System B is not safe” are not the description of events. “Safe” is a kind of degree, and cannot be determined as “true” or "false".

Probability is only given to events. Based on this argument, the “probability of probability” is meaningless. It is possible to say “the probability that its frequency is inside a certain range”.

"State of knowledge” is analyst's belief or experience at a specific time. Analyst can determine “one event is true or false” based on his state of knowledge.

In risk analysis, some events are very rare, so we have to use experts’ opinions. As a result of risk analysis, some probability is obtained, which reflects the state of knowledge of the analyst and it is consistent with all the beliefs of analyst.

In that sense, it is subjective(Personal), but the consistent analysis can be also said as objective. We have low level of knowledge as “degree of belief”. A “degree of belief” can be expressed by “degree of probability”, as probability corresponds to belief. Probability can be measurable quantity, so, belief can be also measurable quantity. "Belief” is imperfect knowledge or mind state of imperfect knowledge. Therefore, knowledge can be measurable quantity. By this way "knowledge" can be quantified to reflect to some probabilistic distribution.

# 3 Bayes' theorem

Richard Price found out an essay; “An Essay Towards Solving a Problem in the Doctrine of Chances”, in Thomas Bayes(1702-1761)’s posthumous and he recognized its importance.

Three years after Bayes’ death, it was published on Philosophical Transactions of the Royal Society, in 1763[3]. The essay greatly influenced to many European philosophers. Laplace applied Bayes’ theorem to many of his works.

In mid 19th century, Boole made counterarguments to Bayes’ theorem. After that, many arguments had broken up about its interpretation, application, and the arguments still continue today.

Bayes' Theorem is a way of understanding how the probability that a theory is true is affected by a new piece of evidence. In the philosophy of science, it has been used to try to clarify the relationship between theory and evidence.

Analyst’s State of Knowledge will change with a new evidence. Bayes’ theorem is a method to treat this change （Updating） with consistency.

Bayes’ fundamental equation is given by the next equation:

$$P_r(\theta_j / HE) = \frac{P_r(\theta_j / H) P_r(E / \theta_j H)}{P_r(E/H)} \cdots (1)$$

Where H: Prior knowledge of analyst, E: New evidence, $\theta_j$: Random variable, $Pr(\theta_j/H)$: Prior distribution, Probability of $\theta_j$ under the condition of knowledge H, $Pr(\theta_j/HE)$: Posterior distribution, Probability of $\theta_j$ under the condition of knowledge H and new evidence E, $Pr(E/H\theta_j)$:Likelihood of new evidence E under the condition of $\theta_j$ and H are given, Pr(E/H) :normalization factor given by the following equation:

$$Pr(E/H)=\Sigma Pr(\theta_j/H) \cdot Pr(E/H\theta_j) \qquad (2)$$

For proving the adequacy of Eq. (1), consider the following relation. For two mutually independent events A, B, the following relations hold. They are pure relations between conditional occurrences of events without any meanings.

$$P(B) = P(A \cap B) + P(\bar{A} \cap B) = P(A)P(B/A) + P(\bar{A})P(B/\bar{A}) \quad (3)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B/A)}{P(B)} = \frac{P(A)P(B/A)}{P(A)P(B/A) + P(\bar{A})P(B/\bar{A})} \quad (4)$$

Here consider the events $A_1$, $A_2$, $A_3$, …. $A_n$. , which are mutually exclusive and have the following relation.
$A_1+A_2+A_3+ \ldots + A_n$ = universal event. (5)

Then,

$$P(A_i/B)=\frac{P(A_i)P(B/A_i)}{P(A_1)P(B/A_1)+P(A_2)P(B/A_2)+\ldots+P(A_n)P(B/A_n)} \qquad (6)$$

If $A_i$ is replaced by $\theta_j$, the form of Eq. (1) is obtained. The relation of (1) is always true. Meaning or interpretation of each term is given, and the Eq. (1) becomes updating method with new evidence (Bayes' equation).

Bayes' equation for continuous distribution becomes as follows.

$$\pi'(\lambda/E)=\frac{\pi(\lambda)L(E/\lambda)}{\int_0^\infty \pi(\lambda)L(E/\lambda)d\lambda} \qquad (7)$$

Where π'(λ/E): posterior distribution, π (λ): prior distribution, L(E/λ):likelihood function, Probability of E's occurrence under the condition λ is given, E: new evidence.

Now, consider concrete expressions of prior and posterior distributions. If we can conduct many experiments, failure rate can be determined as λ for a specific component. But, with few experiments, we cannot determine λ. In this case, it is possible to say that value of λ roughly represents failure rate with the "state of knowledge". It is represented by probabilistic distribution function *π(λ)*, which is prior distribution.

Then, assume that failure rate λ of a component is expressed by a gamma distribution ( α=2: β=2000, Γ(2)= 1 ).

$$\pi(\lambda)=2000^2\lambda\exp(-2000\lambda),$$

since gamma distribution is given by,

$$\pi(\lambda)=\frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}\exp(-\beta\lambda) \qquad (8)$$

Next, "one time failure during 10000hours usage" is observed as a new evidence. The likelihood of this new evidence can be expressed by Poisson distribution which is given by:

$$f(r)=e^{-m}\frac{m^r}{r!} \qquad (9)$$

.

Where failure time "*r*" is 1 and expected number "*m*" equals to 1000λ. Then the likelihood becomes,

$$L(E/\lambda)=10000\lambda\exp(-10000\lambda). \qquad (10)$$

From the Bayes' equation, posterior distribution π'(λ/E) becomes as follows.

$$\pi'(\lambda/E)=\frac{\pi(\lambda)L(E/\lambda)}{\int_0^\infty \pi(\lambda)L(E/\lambda)d\lambda}\ . \qquad (11)$$

Where numerator of Eq. (11) is given by

$$\begin{aligned}\pi(\lambda)L(E/\lambda)&=2000^2\lambda\exp(-2000\lambda)10000\lambda\exp(-10000\lambda)\\&=4\times10^{10}\lambda^2\exp(-12000\lambda).\end{aligned} \qquad (12)$$

Denominator of Eq.(11) is given by

$$\begin{aligned}\int_0^\infty \pi(\lambda)L(E/\lambda)d\lambda&=\int_0^\infty 4\times10^{10}\lambda^2\exp(-12000\lambda)d\lambda\\&=4\times10^{10}\left[\frac{\exp(-12000\lambda)}{-12000}(\lambda^2+\frac{2\lambda}{12000}+\frac{2}{12000^2})\right]_0^\infty\\&=4\times10^{10}\times\frac{2}{12000}\times\frac{1}{12000^2}=4.63\times10^{-2}.\end{aligned} \qquad (13)$$

Finally, posterior distribution becomes,

$$\pi'(E/\lambda)=\frac{12000^3\lambda^2}{2}\exp(-12000\lambda). \qquad (14)$$

The posterior distribution is also gamma distribution with (α=3: β=12000, Γ(3)=2). Mean of prior distribution is given by

$$\overline{\lambda_0}=\int_0^\infty \lambda\pi(\lambda)d\lambda=\int_0^\infty 2000^2\lambda^2\exp(-2000\lambda)d\lambda=1\times10^{-3}. \qquad (15)$$

Mean of posterior distribution is given by,

$$\begin{aligned}\overline{\lambda}&=\int_0^\infty \lambda\pi'(\lambda)d\lambda=\int_0^\infty 12000^3\lambda^3\exp(-12000\lambda)d\lambda\\&=\left[\frac{0.5\cdot12000^3\lambda^3\exp(-12000\lambda)}{-12000}(\lambda^3-\frac{3\lambda^2}{-12000}+\frac{6\lambda}{(-12000)^2}-\frac{6}{(-12000)^3})\right]_0^\infty\\&=2.5\times10^{-4}\end{aligned} \qquad (16)$$

By the additional evidence E, mean of failure rate λ decreases to one-fourth (1/4) of previously expected value.

# 4 Pair of conjugate distributions

Take up the case in which prior distribution is gamma distribution and likelihood is expressed by Poisson distribution. The posterior distribution is calculated by the following steps.

$$\begin{aligned}\pi'(\lambda/E)&=\frac{\pi(\lambda)\cdot L(E/\lambda)}{\int_0^\infty \pi(\lambda)\cdot L(E/\lambda)\,d\lambda}\\&=\frac{\frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}\exp(-\beta\lambda)\cdot\frac{(\lambda T)^k}{k!}e^{-\lambda T}}{\int_0^\infty\frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}\exp(-\beta\lambda)\cdot\frac{(\lambda T)^k}{k!}e^{-\lambda T}\,d\lambda}\\&=\frac{\lambda^{k+\alpha-1}\exp\{-(\beta+T)\lambda\}}{\int_0^\infty\lambda^{k+\alpha-1}\exp\{-(\beta+T)\lambda\}d\lambda}\end{aligned} \qquad (17)$$

By changing the expression of parameters as α'＝ α +k, β'＝β+T, then Eq.(17) is given by

$$\pi'(\lambda/E) = \frac{\lambda^{\alpha'-1}\exp(-\beta'\lambda)}{\int_0^\infty \lambda^{\alpha'-1}\exp(-\beta'\lambda)d\lambda} = \frac{\beta'^{\alpha'}}{\Gamma(\alpha')}\lambda^{\alpha'-1}\exp(-\beta'\lambda) \tag{18}$$

The posterior distribution becomes gamma distribution and the parameter values can be obtained by simple arithmetic calculation. This pair of prior distribution and likelihood is called conjugate[4].

For the convenient usage to the interested readers, several pairs of conjugate distributions are shown below with changing assumptions of prior distribution, likelihood and posterior distribution.

(1) Prior: gamma, Likelihood: gamma
Posterior : gamma, Relation of parameters,
α"=α＋α' −1、 β"=β＋β' /λ*,
where λ* is a specific failure rate value given as a new evidence.

(2) Prior: Beta distribution, Likelihood: binomial distribution
Posterior: Beta, Relation of parameters,
a' = a+k、 b' = b+N－k .

(3) Prior: normal distribution, Likelihood: normal distribution
Posterior: normal distribution, Relation of parameters,

$$\mu' = \frac{\sigma_*^2}{(\sigma_*^2+\sigma_0^2)}\mu_0 + \frac{\sigma_0^2}{(\sigma_*^2+\sigma_0^2)}x_*$$

$$\sigma' = \frac{1}{\sqrt{(\frac{1}{\sigma_*^2}+\frac{1}{\sigma_0^2})}}$$

(4) Prior: log-normal distribution, Likelihood: log-normal distribution
Posterior: log-normal distribution, Relation of parameters,

$$\mu' = \frac{\sigma_*^2}{(\sigma_*^2+\sigma_0^2)}\mu_0 + \frac{\sigma_0^2}{(\sigma_*^2+\sigma_0^2)}\ln x_*$$

$$\sigma' = \frac{1}{\sqrt{(\frac{1}{\sigma_*^2}+\frac{1}{\sigma_0^2})}}$$

# 5 Sequential and lump evaluations

New evidence E1 is obtained, and then posterior distribution π' is evaluated by Bayes' theorem as shown in previous section.

Consider the case that new evidence E2 is further obtained after the first evaluation. In this situation, we can again evaluate posterior distribution π''by the following way.

By the first evidence E1 it becomes

$$\pi'(\lambda/E_1) = \frac{\pi(\lambda)L_1(E_1/\lambda)}{\int_0^\infty \pi(\lambda)L_1(E_1/\lambda)d\lambda}\ . \tag{19}$$

By the second evidence E2 it further becomes

$$\pi''(\lambda/E_1,E_2) = \frac{\pi'(\lambda/E_1)L_2(E_2/E_1,\lambda)}{\int_0^\infty \pi'(\lambda/E_1)L_2(E_2/E_1,\lambda)d\lambda}\ . \tag{20}$$

Substitute the first Eq.(19) into the second Eq.(20), we obtain:

$$\pi''(\lambda/E_1,E_2) = \frac{\pi(\lambda)L_1(E_1/\lambda)L_2(E_2/E_1,\lambda)}{\int_0^\infty \pi(\lambda)L_1(E_1/\lambda)d\lambda \frac{\int_0^\infty \pi(\lambda)L_1(E_1/\lambda)L_2(E_2/E_1,\lambda)d\lambda}{\int_0^\infty \pi(\lambda)L_1(E_1/\lambda)d\lambda}}$$

$$= \frac{\pi(\lambda)L_1(E_1/\lambda)L_2(E_2/E_1,\lambda)}{\int_0^\infty \pi(\lambda)L_1(E_1/\lambda)L_2(E_2/E_1,\lambda)d\lambda}\ . \tag{21}$$

Use the following relation between conditional occurrences of events.

$$L_1(E_1/\lambda)L_2(E_2/E_1,\lambda)=L(E_2,E_1/\lambda) \tag{22}$$

*L(E₂,E₁/λ)* can be interpreted as a likelihood in which two evidences $E_1$ and $E_2$ are combined and occur at the same time. Then Eq.(21) becomes

$$\pi''(\lambda/E_1,E_2) = \frac{\pi(\lambda)L(E_1,E_2/\lambda)}{\int_0^\infty \pi(\lambda)L(E_1,E_2/\lambda)d\lambda}\ . \tag{23}$$

The above Eq.(23) is the result of lump (one step) evaluation by the Bayes' method. Eq. (21) equals to Eq. (23). This means that exactly the same result is obtained from the sequential (two steps) evaluation or lump (one step) evaluation for the existence of two evidences by Bayes' method.

# 6 Application of the Bayes' theorem

## 6.1 Data evaluation by Bayes' theorem

The procedure of data evaluation by Bayes' theorem is performed by the procedure as shown in the following for a specific plant risk evaluation:
(1) Determine prior distribution,
(2) Get operating experience of a specific plant,
(3) Determine likelihood function, and finally
(4) Obtain posterior distribution by Bayes' theorem.
For the determination of prior distribution, some judgment is required by analysts, because many data sources do not clearly describe data conditions such as failure modes, environmental condition, produced company, *etc*. Several example practices for nuclear

applications are introduced in the subsequent sections in this chapter.

6.1.1 Start of Diesel generator

The data of Reactor Safety Study (RSS)[1] are used for the determination of prior distribution. The RSS gives the median value as 0.03/demand with error factor 3 for the failure probability of "Start of Diesel generator" when it is demanded to work.

Let us try to express the prior distribution by Log-normal distribution. Failure probability per demand is assumed to be expressed by the notation $Q$.

The parameters and key statistical values are adjusted as follows.

$Q_{05}=10^{-2}$

$Q_{95}=10^{-1}$

Median：$m_\alpha=SQRT(Q_{05}xQ_{95})=SQRT(10^{-3})=0.032$

$\mu=\ln(m_\alpha)=\ln(0.032)=-3.442$

$\sigma = (\ln Q_{95}-\ln Q_{05})/(1.64x2)$

$\quad = (-2.302-(-4.605))/3.28 = 0.702$

Mean = $\exp(\mu+\sigma^2/2) = \exp(-3.442+0.2464)$

$\quad = \exp(-3.1956) = 0.041$

Variance = $\exp(2\mu+\sigma^2)[\exp(\sigma^2)-1]$

$\quad = \exp(-6.884+0.4928)\ [1.637-1] = 0.0011$

Then, prior distribution becomes,

$$\pi(Q)=\frac{1}{\sqrt{2\pi}\sigma Q}\exp(-\frac{(\ln Q-\mu)^2}{2\sigma^2})$$
$$=\frac{1}{1.7597Q}\exp(-\frac{(\ln Q-3.442)^2}{0.9856})\ . \tag{24}$$

As plant specific data, there exist 4 times of failure among total 300 times of start tests. The likelihood of this evidence becomes as follows.

$$L(E/Q_i)={}_nC_rQ_i^r(1-Q_i)^{n-r}=\frac{n!}{r!(n-r)!}Q_i^r(1-Q_i)^{n-r}$$
$$=\frac{300\cdot 299\cdot 298\cdot 297}{4\cdot 3\cdot 2\cdot 1}Q_i^4(1-Q_i)^{296}=3.308\times 10^9 Q_i^4(1-Q_i)^{296} \tag{25}$$

The prior and posterior distributions are shown in Figure 1.

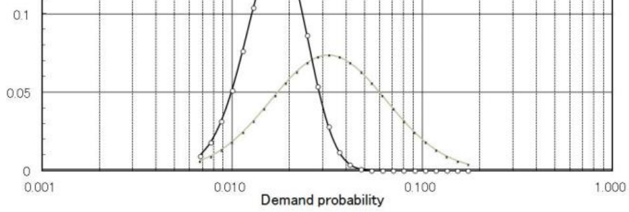


Fig. 1 Prior and posterior distributions for start of DG.

In this present case, prior distribution and likelihood are not a pair of conjugate distributions. So, the posterior distribution is obtained by Excel table, in which numerical calculations are performed for all the $Q_i$ from 0.006 to 0.17.

The values of mean and variance are easily obtained from Excel table. Mean value of posterior distribution decreases to 0.018 and variance also decreases to $4.1x10^{-5}$. Smaller mean value and more narrow distribution of demand probability of the start of DG is obtained by the operational records of a plant.

6.1.2 Failure rate of pressure sensor

Failure rate of pressure sensor is given by ANSI/ IEEE Std-500 [5] as follows:

| | |
|---|---|
| maximum value | 40 $x10^{-6}$/h |
| recommended value | 1.12 $x10^{-6}$/h |
| minimum value | 0.03 $x10^{-6}$/h |

Above three values are considered as $\lambda_{95}$, $\lambda_{50}=m_\alpha$, $\lambda_{05}$, respectively. Then, error factor becomes

$$\sqrt{\lambda_{95}/\lambda_{05}}=\sqrt{\frac{40\times 10^{-6}}{0.03\times 10^{-6}}}=36.5\ . \tag{26}$$

From this value, $\lambda_{95}$ and $\lambda_{05}$ are re-evaluated as Follows, respectively;

$$\lambda_{95}=36.5\times m_\alpha=36.5\cdot 1.12\times 10^{-6}/h=40.9\times 10^{-6}/h$$
$$\lambda_{05}=m_\alpha/36.5=1.12\times 10^{-6}/36.5=0.0307\times 10^{-6}/h$$

The above results are reasonable by comparing with their original values. The prior distribution is assumed as Log-normal distribution. Then, parameter values become as follows;

$$\mu=\ln m_\alpha=\ln 1.12\times 10^{-6}=-13.702$$
$$\sigma=(\ln\lambda_{95}-\ln m_\alpha)/1.64$$
$$=(-10.1044+13.702)/1.64=2.194$$
$$mean:\alpha=\exp(\mu+\frac{\sigma^2}{2})=\exp(-13.702+2.406)$$
$$=1.24\times 10^{-5}/h$$
$$\mathrm{var}iance:\beta^2=\exp(2\mu+\sigma^2)\left\{\exp(\sigma^2)-1\right\}$$
$$=\exp(-22.592)(\exp(4.814)-1)=1.8\times 10^{-8}$$

Prior distribution becomes as follows;

$$\pi(\lambda)=\frac{1}{\sqrt{2\pi}2.194\lambda}\exp(-\frac{(\ln\lambda+13.702)^2}{9.63})\ . \tag{27}$$

It is observed in a specific plant that 5 failures occur during total $1.5x10^5$ hours operation. Poisson distribution is assumed for the likelihood function. Expected failure time ($m$) becomes $\lambda_i T$, and observed failure time $r$ is 5. The likelihood function is given by the following Eq. (28).

$$L(E/\lambda_i)=e^{-\lambda_i T}\frac{(\lambda_i T)^r}{r!}=e^{-1.5\times10^5\lambda_i}\frac{(1.5\times10^5\lambda_i)^5}{5!} \quad . \quad (28)$$

Posterior distribution is also obtained by Excel table as shown in Figure 2.

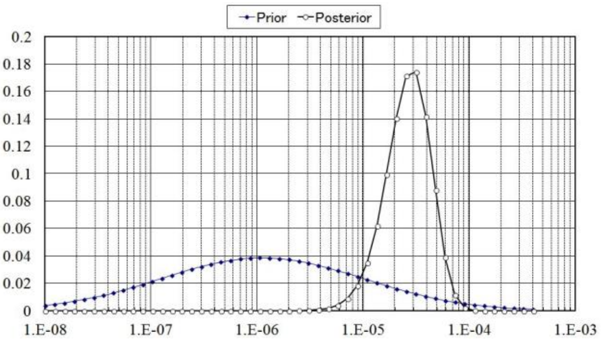


Fig. 2 Prior and posterior distributions of failure rates of pressure sensor.

From the Excel table, the following parameter values are obtained.

$$mean:\alpha=2.90\times10^{-5}/h$$

$$\mathrm{var}\,iance:\beta^2=5.10\times10^{-10}/h$$

**6.2 Frequency of nuclear power plant accidents**

Occurrence frequency of any specific accidents in nuclear power plants can be evaluated by Bayes' method by the following ways: First suppose prior distribution. Then collect accident related cases, and these are used as new evidence. Several example practices for nuclear applications are introduced in the subsequent sections in this section.

6.2.1 Fires in nuclear reactor compartment

There are very few data about fires in nuclear power plants. As prior distribution, assume gamma distribution.

$$\pi(\lambda)=\frac{\beta^\alpha}{\Gamma(\alpha)}\ \lambda^{\alpha-1}\ \exp(-\beta\lambda);\alpha=0.32,\beta=0.29 \qquad (29)$$

Above distribution has very wide range.

$$\lambda_{05}=2.1\times10^{-4},\ \ \lambda_{50}=0.30,\ \ \lambda_{95}=5.0,$$
$$mean=\alpha/\beta=1.11/reactor\cdot year$$

Likelihood function is assumed as Poisson distribution as below, because evidence may be few events.

$$L(E/\lambda_i)=\frac{(\lambda_i T)^k}{k!}e^{-\lambda_i T} \qquad (30)$$

We assume that the collected evidence is that 10 fire cases occur during 300 reactor years. Parameters of posterior distribution become as follows,

$$\alpha'=\alpha+k=0.32+10=10.32,$$
$$\beta'=\beta+T=0.29+300=300.29$$
$$\lambda_{05}=1.8\times10^{-2},$$
$$\lambda_{50}=3.3\times10^{-2},$$
$$\lambda_{95}=5.4\times10^{-2},$$
$$mean=\alpha'/\beta'=10.32/300.29=3.44\times10^{-2}\ /reactor\cdot year$$

Above parameter values are obtained by the relation of conjugate distributions as shown in Chapter 4.

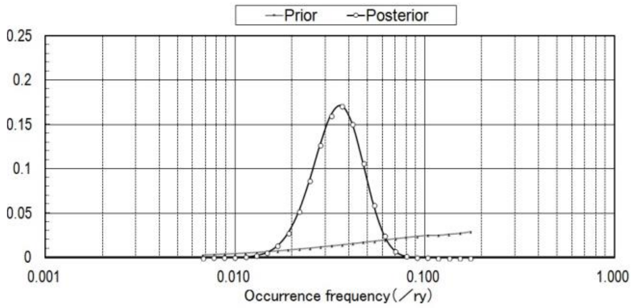


Fig. 3 Occurrence frequency of fires in nuclear power plants.

6.2.2 Core melt frequency with the experience of Fukushima Dai-ichi accidents

At the Fukushima Dai-ichi accidents, three core melt accidents had happened. In Japan, there are about 50 nuclear reactors and experienced operational time period is estimated as 1003 reactor·year until 2012. Then, core melt frequency is estimated as 2.99x$10^{-3}$/r·y.

If we assume only three times of core melt accidents are very lucky cases, then put the following value equals to 0.01.

$$\sum_{n=0}^{3}\frac{(\lambda T)^n}{n!}\cdot\exp(-\lambda T)=\left(1+\lambda T+\frac{(\lambda T)^2}{2}+\frac{(\lambda T)^3}{6}\right)\exp(-\lambda T) \qquad (31)$$

Occurrence frequency is calculated as, *$\lambda T$=10.045*, $\lambda$=1x$10^{-2}$/r·y. The above value is considered as the upper limit of core melt frequency $\lambda_{99}$=1x$10^{-2}$/r·y.

In Japan, there are several near-miss accidents as follows which might cause core melt in the past.

-Takahama (1979.11.3); Leakage of primary coolant,
-Fukushima Dai-ni (1989.1.6);Failure of recirculation pump,
-Mihama(1991.2.9); Steam generator tube rapture,
-Tsuruga(1999.7.12); Leakage of primary coolant,
-Hamaoka(2001.11.7); Explosion of hydrogen gas, and
-Fukushima Dai-ichi (2011.3.11); Tsunami caused also the units 5 and 6 almost no heat sink and station power.

Those accidents are counted as 0.7 times of core melt accidents. Then, total 3.7 times core melt accidents happened in Japan. This value is considered as $\lambda_{50}$=3.7x10$^{-3}$ / r y.

As prior distribution, parameters are adjusted as follows by using Gamma distribution.

mean = $\alpha/\beta$ =3.7x10$^{-3}$

$\alpha$=3.29, $\beta$= 889, for $\lambda_{99}$=1x10$^{-2}$

median = 3.33x10$^{-3}$

Here consider an expert's opinion. Professor Rasmussen said $\lambda_E$=1.5x10$^{-5}$/r y. Take this opinion as new evidence and the likelihood function is assumed as the following form.

$$L(\lambda_E/\lambda) = L(\lambda_E/\lambda,\nu,K) = \frac{K^{\nu K+1}}{\Gamma(\nu K+1)}\left(\frac{\lambda}{\lambda_E}\right)^{\nu K}\cdot \exp\left(-K\left(\frac{\lambda}{\lambda_E}\right)\right) \tag{32}$$

Where parameter $\nu$ is the value how we believe the expert opinion. In this case, $\nu$ is put as 50, which means that we evaluate that Rasmussen predicted 50 times smaller value from the true value (which could be existed.). In Eq.(32), K = $\lambda_E$ T is times of core melt accidents predicted by experts. The likelihood (Eq. (32)) reflects the expert opinion and analyst's judgment.

As the prior and likelihood are pair of gamma distributions, posterior distribution becomes gamma distribution, and parameters are deduced as follows,

mean = $\alpha'/\beta'$ =1.2x10$^{-3}$/ r y

$\alpha'$ =6.99, $\beta'$ = 5822

Our believed value of core melt frequency thus becomes 1.2x10$^{-3}$/r y by considering Professor Rasmussen's opinion.

# 7 Discussions between favorable and critical peoples to the Bayes' methods

Arguments still continue among favorable and critical peoples about Bayes' theorem. Their opinions can be briefly summarized as follows by citing several views and arguments from Ref. [6].

It is not clear that prior distribution certainly exists, but it is used so easily. Even if it exists, there is not any method to identify it precisely. From above reasons, it is not possible to keep objectivity for the result obtained by Bayes' method. On the contrary, the estimation based on the sampling method does not bring subjectivity to the theory [7].

Both from pro-Bayesian and contra-Bayesian or non-Bayesian analysis we have to guess that some mathematical model is adequate one in reality. The main difference is that in a non-Bayesian analysis more important something may be swept away under the carpet [8].

Engineers who represent their degree-of-belief by probability must be stout-hearted [9].

There can be no question that it is extremely difficult to determine a person's utility function even under the most ideal and idealized experimental conditions: one can almost say that it has yet to be done [10].

I would like to object to the statement, repeatedly made, that *a priori* is unknown. It is ridiculous to say that *a priori* is a statement of one's knowledge, and a modern work demonstrates that it is always known by judicious questioning it can be found [11].

# 8 Concluding remarks

Bayesian methods provide a logical framework for safety analysis of nuclear power plants. The frequentist methods seem to be objective, but they limit the available evidence to the only statistical values. In actual situation, however, we have to make judgment to decide what the evidence is. Furthermore, our knowledge about nuclear power plant is formed not only by statistical data but also from design considerations, operating environment and so on. Bayesian methods can translate these beliefs into numbers, and assessors become coherent. Then the group of assessors has high chances to reach a common decision.

It is the author's hope that readers find out usefulness of Bayes' theorem explained here, and utilize it for your future research activities.

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