

A method to detect and fix response delay of sensors measurement in sinusoidal flow condition

ZHOU Bao¹, and GAO Puzhen²

1. Fundamental Science on Nuclear Safety and Simulation Technology Laboratory, Harbin Engineering University, 145Nantong Street, Harbin 150001, China (zhou0123bao@163.com)

2. Fundamental Science on Nuclear Safety and Simulation Technology Laboratory, Harbin Engineering University, 145Nantong Street, Harbin 150001, China (gaopuzhen@sina.com)

Abstract: In this paper, single-phase experiments were conducted in different sinusoidal flow condition for studying the dynamic response of measurement sensors. The obtained experimental data were fitted as the form of transfer function between flow rate signal as Input and pressure drop along the channel as Output of the dynamical system and to give the transfer function model of the second order model. By the derived transfer functions it becomes possible to trace the actual flow condition in the channel, by which it becomes possible to measure magnitude deviation and the phase difference caused by the sensors' natural dynamic response delay.

Keyword: sensor delay, phase difference, magnitude deviation, sinusoidal change of channel flow

1 Introduction

There arises various kinds of error to measure any parameters in any experiment. One kind of error is caused by the response delay of the sensors when measuring flow. This problem can be neglected in steady state condition when only the average value is in concern. But when the instantaneous value is needed to measure as precisely as possible, it is not so easy as to measure it for steady state value. For example, when the pressure value in the flow channel jumps sharply from 0 to 1, the measured pressure value would not be like a sharp jump but a smooth curve gradually approaching to the final value as shown in Fig. 1.

In case of changing flow as sinusoidal wave, the measured value would be, as seen in Fig.2, both amplitude changing and phase difference. This problem of instantaneous value measurement will bring the mismatching of the flow rate data with the pressure drop data in the flow channel experiment with dynamically changing flow condition. From the practical point of view, it will become a safety issue for any industrial system. For example, the sensors for temperature, pressure and flow rate setting in nuclear power plant should provide relatively precise instantaneous signal to ensure that the automation control system to measure their correct values and to

make timely actions in order to avoid serious situations such as nuclear fuel burn-out. Therefore, it is essential to understand the nature of measurement errors to be taken into consideration in safety design.

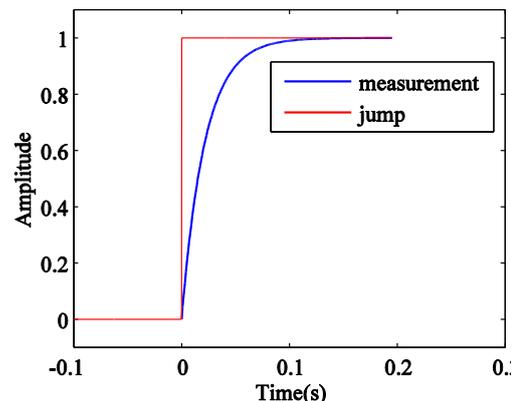


Fig. 1 Step input and its output.

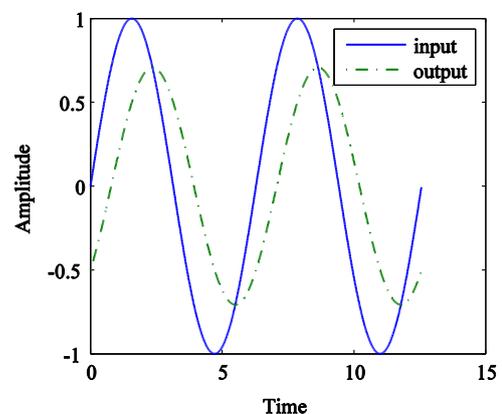


Fig. 2 Sinusoidal input and its output.

However, it is difficult to clarify the nature of measurement error solely by theoretical method,

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because details of the entire measure system such as sensor material, geometry, quality condition, connection condition, installation and environmental condition. Moreover some of these conditions may be changing during operation.

As an alternative way to solve this issue, we can apply an arbitrary equation model for the theoretical analysis to be able to detect the behavior of the sensor. The general model comes from the first-order, linear time-invariant (LTI) dynamic response system^[6]. As an example, a simple heat transfer problem^[12] is introduced as below,

$$\tau \frac{dO(t)}{dt} = I(t) - O(t) \quad (1)$$

where I and O : functions of time t , and τ : Exponential Decay Constant. The term I is regarded as the system input (force function describing an external driving function of time), while O is the response, or system output. By conducting Laplace formation for Eq.(1), we get

$$G(s) = \frac{O(s)}{I(s)} = \frac{1/\tau}{1/\tau + s} \quad (2)$$

where $G(s)$ is called transfer function.

In case of step input signal I given by

$$I(t) = \begin{cases} 0, & t < 0 \\ A_m, & t \geq 0 \end{cases} \quad (3)$$

the obtained Laplace formation is A_m/s , and so

$$O(s) = \frac{A_m}{s} \times \frac{1/\tau}{1/\tau + s} \quad (4)$$

By the inverse Laplace transformation of Eq.(4), we get

$$O(t) = A_m(1 - e^{-\frac{t}{\tau}}) \quad (5)$$

where τ is defined as Time Constant which means how long time the output value needed to become 63% of the input value. And its long time solution is

$$O(t) = A_m \quad (6)$$

For sinusoidal input,

$$I(t) = A_m \sin(\omega t) \quad (7)$$

The long time solution is

$$O(t) = \frac{A_m \sin(\omega t - \psi)}{\sqrt{1 + (\omega\tau)^2}} \quad (8)$$

where

$$\tan(\psi) = \omega\tau \quad (9)$$

As seen in Eq. (8), the amplitude will vary and there will be phase difference between I and O .

The above example is the first order system. While the system to be studied is not necessarily has the

same order. In this paper, we mainly discuss on the variation of amplitude and the phase difference by conducting an experimental research where sinusoidal wave is added on the part of input signal while observing the output signal in the experimental set up.

The target experiment is related with a simple thermal-hydraulic test section and the sensors equipped in the test section are both flow rate and pressure drop along the test channel. Gao Pu-zhen^[5] *etc.* studied the pressure drop of single-phase sinusoidal fluctuation flow in a circular cross section pipe of the diameter 16mm. They treated the delay time problem simply by counting the difference between the two starting point of the data, while errors in n examples was relatively large to be compared with both their theoretical analysis on laminar flow condition and the analytic results by CFD (Computational Fluid Dynamics). However, they did not consider the dynamic response.

In the area of automatic control, researchers discuss on this sensor problem in detail both in theoretical and experimental studies^[12] where the usage of reference sensor is assumed. But for authors of this paper it is difficult to have a reference sensor to give the exact condition of input signal. It is also impractical to disassemble the meters used in the authors' test section. Although the instructions of some meters give the time constant values, most of them are for step input system, also it is hardly in good agreement with the different operating situation, unless we can make certain that it fits with the first order system. Therefore in this paper an experimental study was conducted on sinusoidal flow in different flow conditions to fit the experimental data with the dynamic response system transfer function in order to detect the variation of amplitude and the phase difference of the Flow Meter and Differential Pressure Transmitter.

2 Experimental system

The experimental system employed in this study is illustrated in Fig. 3. As is shown in Fig. 3, it consists of two parts, the experimental loop and the Data Acquisition System. In Fig. 3, the solid line represents the pipe connection, while the dashed line, the circuit

connection, and the details of the two connections are explained in 3.1 and 3.2, respectively.

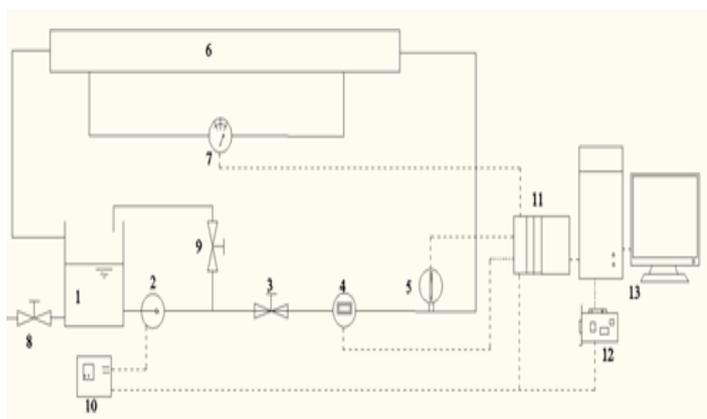


Fig. 3 Experimental loop.

- 1 Tank, 2 Pump, 3 Valve, 4 Flow Meter,
- 5 Thermometer, 6 Test Section, 7 Differential Pressure Transmitter, 8 Valve, 9 Valve, 10 Inverter, 11 NI DAQ, 12 PCI-1711, 13 PC

3.1 Pipe connection

The water is pumped from the tank, and it flows through Flow Meter, thermometer, test section and back to the tank to form a flow loop. The size of test section is 40mm×3mm of Width ×Height in rectangular channel. There are two pressure taps on it, connecting to the Differential Pressure Transmitter, whose distance is 1.4m. The pump power is controlled by an Inverter. The control signal is 0-5 V analog voltage signal, which comes from a PCI-Lab Card PCI-1711. This card is programmed to collect digital sinusoidal signal and convert it to analog sinusoidal signal.

3.2 The Data Acquisition System

All of the signals, flow rate signal, pressure drop signal, temperature signal and control signal are collected by National Instruments Data Acquisition System card with the collection frequency of 50 Hz.

3 Pre-experiment

To test the reliability of entire experimental system, steady state flow experiments were carried out in beforehand. The comparison of the experimental data with empirical formulas can be seen in Fig. 4, where C_1 is a constant value for rectangular channel, which defined by Shah and London ^[2] (for the general pipe which should be $\lambda=64/Re$); The constant C_1 is defined

by Sadatomi (1982) ^[3], the experimental data both in the laminar region and the turbulent region agrees well with the empirical equations which indicates the Differential Pressure Transmitter and the Flow Meter works fine in steady state.

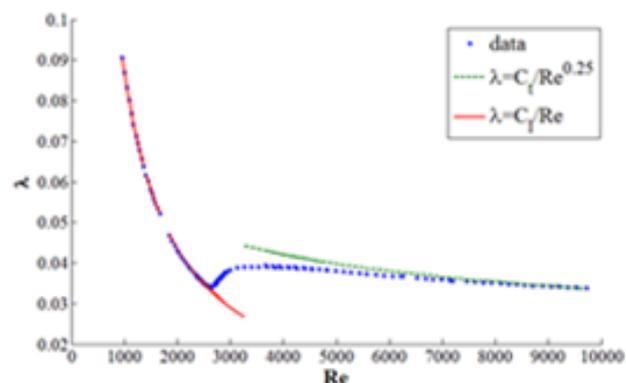


Fig. 4 Comparison of experimental data with empirical formula.

4 Calculation of the phase difference and the variation of magnitude

4.1 Experimental procedure

Totally 173 cases of single phase sinusoidal flow experiment had been conducted with the varying period of 6.26S, 10.95S, 15.64S, 20.33S, 30.50S and 40.67S, respectively, and also with average Reynolds number from 1292 to 11577 (Reynolds number amplitude from 486 to 6013).

The experimental procedure is shown in Fig. 5. First, input the parameters of digital sinusoidal signal such as the average, the amplitude and the period to the Control Program interface. This digital signal is collected and converted to analog signal. The Inverter amplify the signal to change rotating speed of the pump, and then the pump push the water.

4.2 Calculation procedure

Besides collecting the pressure drop and flow rate data, the control signals were collected at same time of which expression is given by Eq. (11). This is for setting benchmarking signal from which all the values of the dynamic input signal and the output signals are obtained. Both the obtained data of pressure drop data and flow rate data are explained in 4.2.1 and 4.2.2, respectively, as below.

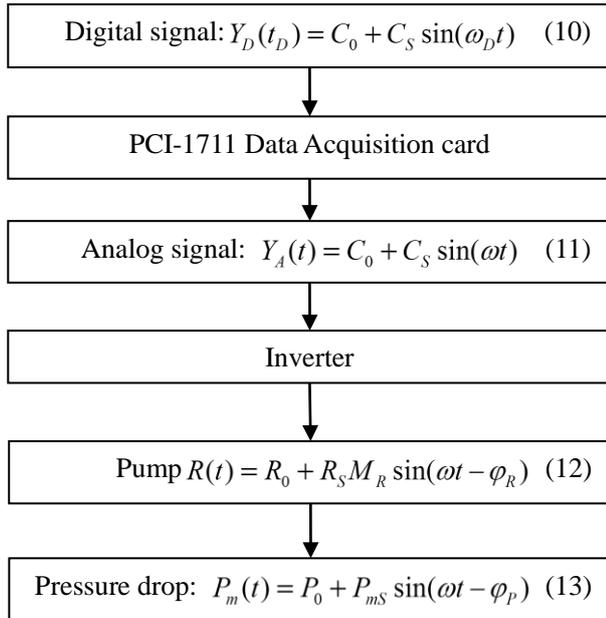


Fig. 5 Experimental flow chart.

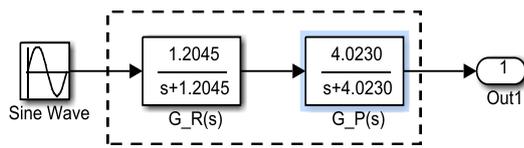


Fig. 6 Pressure drop's dynamic I/O system.

4.2.1 Pressure drop data

As can be seen in Fig. 6, there are two transfer procedures between the input signal and the pressure drop output signal. They are:

- (a) Transfer procedure from control signal to pump rotation speed, and
- (b) Transfer procedure from pump rotation speed to output signal.

The first one (a) is that the control signal is turned to the pump's rotating speed. This can be seen as a dynamic system^[13] that notwithstanding the setting of a constant target rotating speed, the pump will not reach it at once but gradually reach the steady state constant speed just like Fig. 1 shows. So the rotating speed of the pump has amplitude change and time difference with the control signal as given by Eq. (12).

Changing the pump rotating speed can change the pressure of water, and this causes the pressure propagation with speed of sound, although it can be neglected in the authors' experimental system. The pump rotating speed has square relation with the

pressure value, which is the same with the pressure drop.

$$P(t) = k_R R^2(t) = k_R \left[R_0^2 + 2R_0 R_S M_R \sin(\omega t - \phi_R) + R_S^2 M_R^2 \sin^2(\omega t - \phi_R) \right] \quad (14)$$

The second transfer procedure (b) is that the pressure drop of the water in the channel turn into the output signal coming from the Differential Pressure transmitter. It can be also regarded as a dynamic response system. So the input signal for the Pressure Transmitter is given by

$$P(t) = P_0 + P_S \sin(\omega t - \phi_R) + P_{S2} \sin^2(\omega t - \phi_R) \quad (15)$$

The second order item in Eq.(15) can be neglected because the relation $R_S \ll R_0$ holds in most of the authors' experimental cases and it makes simplify the problem. Also when fitting the experimental data with both 1st order and the 2nd order Fourier series, the authors of this paper found that the differences are minor, although the second order is more precise than the first order. So the input signal for the Pressure Transmitter is given by

$$P(t) = P_0 + P_S \sin(\omega t - \phi_R) = k_R \left[R_0^2 + 2R_0 R_S M_R \sin(\omega t - \phi_R) \right] \quad (16)$$

Therefore, both the input and output signal should be given by the following equations by being normalized by the signals with removing the average item first:

$$I_{pm}(t) = 2 \frac{R_S}{R_0} M_R \sin(\omega t - \phi_R) \quad (17)$$

$$O_p(t) = \frac{P_{mS}}{P_0} \sin(\omega t - \phi_p) \quad (18)$$

It is difficult for the authors of this paper to determine the exact condition of ump's rotating speed $R(t)$ as a function of time t to decide both R_S and R_0 . But the authors of this paper know that the control signal has proportional relationship with the target pump's rotating speeds in steady state, and therefore they assume the following relation

$$\frac{C_S}{C_0} = \frac{R_S}{R_0} \quad (19)$$

This means that the authors of this paper look upon $C(t)$ as the input signal instead, and take this relation between pump's rotating speed and the pressure into consideration, and consider the two dynamic system procedure as one. The input signal is assumed to be given by

$$I_p(t) = 2 \frac{C_s}{C_0} \sin(\omega t) \quad (20)$$

because the authors of this paper do not know what order of the dynamic response systems is. They use MATLAB dynamic system as the black box tools to fit the experimental data of the authors of this paper. For doing this the authors of this paper have to guess how many zero points and how many poles for the Laplace transfer function. It is certain that it has two poles at least, as the input signal has to go at least through two dynamic systems. So the authors of this paper tried from 2 to 5 poles and 0 to 4 zero points, among which the 2 poles and 0 zero points one has reasonable results, because the accuracy in this case is more than 95%. And more poles with more zero points cannot necessarily improve this fitting accuracy. So the authors of this paper got the transfer function as given by the following equations,

$$G_p(s) = \frac{4.51}{s^2 + 5.227s + 4.845} \quad (21)$$

$$G_p(s) \approx \frac{4.0230}{s + 4.0230} \times \frac{1.2045}{s + 1.2045} \quad (22)$$

Considering the formation of this transfer function, it can be divided into two first order systems as is given by Eq.(22), where one is for the pump's transfer system while the other one for the Pressure Transmitter's system. The authors of this paper checked the time constant of Pressure Transmitter in the instruction manual, where it indicates that the setting in the factory is 0.2s. By using these two transfer function, the authors of this paper calculated the time constant for step response. So the authors of this paper pick the closer value in the instruction manual, as the Pressure Transmitter's transfer function, while the other one as the pump's transfer function. The resultant equations are given by

$$G_{pm}(s) \approx \frac{4.0230}{s + 4.0230} \quad (23)$$

$$G_R(s) \approx \frac{1.2045}{s + 1.2045} \quad (24)$$

For Eq. (23), they got the time constant 0.25s. For Eq. (24) they got the time constant 0.84s.

4.2.2 Flow rate data

As for the flow rate data, it should have a proportional relationship with the pump's rotating speed in steady state. The authors of this paper found that the data fit with the sinusoidal equation (first order Fourier series) very well. So they have the output signal for the flow rate:

$$O_Q(t) = \frac{Q_{ms}}{Q_0} \sin(\omega t - \varphi_Q) \quad (25)$$

It has three transfer procedures from the input signal to the output signal. The 1st one is the control signal turning into the pump's rotating speed, the 2nd one is the pressure changing the flow rate, and the 3rd one is the water flow rate turning into the output signal which is measured by the Flow Meter.

Among those three transfer procedures mentioned above, the 2nd one can be determined in advance by the following way.

Consider the pressure drop given by

$$P(t) = P_f(t) + P_a(t) \quad (26)$$

where $P_f(t)$ is friction pressure and $P_a(t)$ is acceleration pressure.

Acceleration pressure $P_a(t)$ is given by

$$P_a(t) = L \frac{dG_t(t)}{dt} = \frac{\rho L}{A} \frac{dQ(t)}{dt} \quad (27)$$

where L : distance between the two pressure taps and G_t : mass flow density ($\text{kg/m}^2 \text{ s}$).

If the flow rate of water is given by

$$Q(t) = Q_0 + Q_s \sin(\omega t - \varphi_x) \quad (28)$$

then the friction should have the same change with the flow rate, because the friction pressure comes from the water flow. So we can set

$$P(t) = P_{f0} + P_{fs} \sin(\omega t - \varphi_x) + \frac{\rho L}{A} Q_s \omega \cos(\omega t - \varphi_x) \quad (29)$$

Here we set

$$P_{as} = \frac{\rho L}{A} Q_s \omega \quad (30)$$

Then we get

$$P(t) = P_{f0} + \sqrt{P_{fs}^2 + P_{as}^2} \sin(\omega t - \varphi_x + \varphi_a) \quad (31)$$

where

$$\sin(\varphi_a) = \frac{P_{as}}{\sqrt{P_{fs}^2 + P_{as}^2}} \quad (32)$$

Compare Eq. (31) with Eq. (16) then we obtain

$$P_{f0} = P_0 \quad (33)$$

$$\sin(\varphi_a) = \frac{P_{as}}{P_s} \quad (34)$$

$$\varphi_x = \varphi_a + \varphi_R \quad (35)$$

So the input signal for the Flow Meter should be given by

$$Q(t) = Q_0 + Q_s \sin(\omega t - \varphi_R - \varphi_a) \quad (36)$$

Or it can be given by

$$Q(t) = Q_0 + Q_s \cos(\varphi_a) \sin(\omega t - \varphi_R) - Q_s \sin(\varphi_a) \cos(\omega t - \varphi_R) \quad (37)$$

In Eq. (37), the first 2 items should have proportional relationship with the pump's rotating speed, so that it has square relationship with the pressure drop. So we have

$$\frac{Q_0^2}{P_0} = \frac{2Q_0Q_s \cos(\varphi_a)}{P_s} \quad (38)$$

Together with Eqs. (21), (32) and (36) we can obtain

$$\sin(2\varphi_a) = \frac{\rho L Q_0}{A P_0} \omega \quad (39)$$

So we can change the output signal as

$$O_Q(t) = \frac{Q_{ms}}{Q_0} \cos(\varphi_a) \sin(\omega t - \varphi_Q + \varphi_a) \quad (40)$$

Thus the number of transfer procedures decreases from 3 to 2, and the remaining ones are for the pump and the Flow Meter with the corresponding input signal given by

$$I_Q(t) = \frac{C_s}{C_0} \sin(\omega t) \quad (41)$$

Using the same approach to fit the flow rate data, we can get the transfer function with its accuracy being more than 95%.

$$G_Q(s) = \frac{1.504}{s^3 + 3.695s^2 + 4.042s + 1.306} \quad (42)$$

$$= \frac{1.504}{(s + 2.0029)(s + 1.0988)(s + 0.5935)}$$

So the Flow Meter's transfer function is given by

$$G_{Qm}(s) = \frac{G_Q(s)}{G_R(s)} \quad (43)$$

So far the author of this paper can obtain the pressure drop and flow rate, by Laplace transform and its inverse transform from the authors' data as output;

$$P(t) = La^{-1} \frac{La(P_m(t))}{G_{Pm}(s)} \quad (44)$$

$$Q(t) = La^{-1} \frac{La(Q_m(t))}{G_{Qm}(s)} \quad (45)$$

Or from the input

$$P(t) = P_0 + P_0 \times La^{-1}[La(I_p(t)) \times G_R] \quad (46)$$

The same way to obtain $Q(t)$ as for the pressure from the input, with being that the amplitude need to be divide by $\cos(\varphi_a)$, the phase need to be minus φ_a .

The factor $(s+1.0988)$ in the denominator of Eq. (42) is close to the denominator of the pump's transfer function's, Eq. (24), which can verify the authors' choice on which one is the pump's and which one is the Pressure Transmitter's.

5 Results

Figure 7 shows measured pressure amplitude data versus corrected pressure amplitude data, while Fig. 8, measured flow rate amplitude data versus corrected flow rate amplitude data. Figure 9 shows the calculated phase angle caused by sensor's dynamic response. Figure 10 shows Bode diagram for the Flow Meter.

Figures 7, 8 and 9 show how different of measured data from the revised data. It can be seen that for the pressure drop data, the amplitude would vary between 85% and 100%. The flow rate data's amplitude would vary from 45% to 130%. The pressure drop's phase angle is calculated to be less than 20°, while the flow rate's phase angle from ca.20° to ca. 90°. The pressure drop's data seems to be acceptable, while the flow rate's data not. So the authors of this paper reconsidered the Flow Meter's transfer function $G_{Qm}(s)$ by the following ways.

$$G_{Qm}(s) = \frac{G_Q(s)}{G_R(s)}$$

$$= \frac{1.504}{s^3 + 3.695s^2 + 4.042s + 1.306} \bigg/ \frac{1.2045}{s + 1.2045} \quad (47)$$

$$= \frac{1}{0.8008s^2 + 1.9945s + 0.8346 + \frac{0.0406}{s + 1.2045}}$$

Neglect the last item $0.0406/(s+1.2045)$ in the denominator of Eq. (47), then Eq. (47) is written as

$$G_{Qs}(s) = \frac{1}{0.8008s^2 + 1.9945s + 0.8346} \quad (48)$$

This Eq.(48) is the typical 2nd order dynamic response system, which is similar to the mass-spring-damper system [12] whose transfer function is given by

$$G(s) = \frac{1}{ms^2 + cs + k} \quad (49)$$

In this system of Eq.(49), the performance depends on two parameter ζ and ω_n . The relation of the transfer function's coefficients to ζ and ω_n is given by

$$\omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = 2 \frac{c}{\sqrt{mk}}$$

For G_{Q_s} given by Eq. (48), $\zeta = 1.2198$ and $\omega_n=1.0209$.

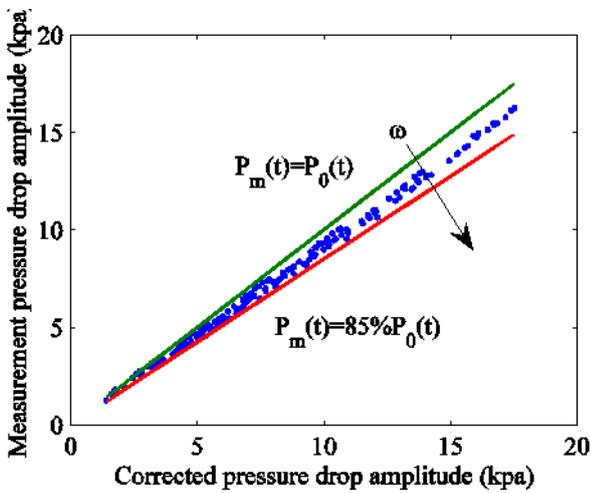


Fig. 7 Measured pressure amplitude data vs. corrected pressure amplitude data.

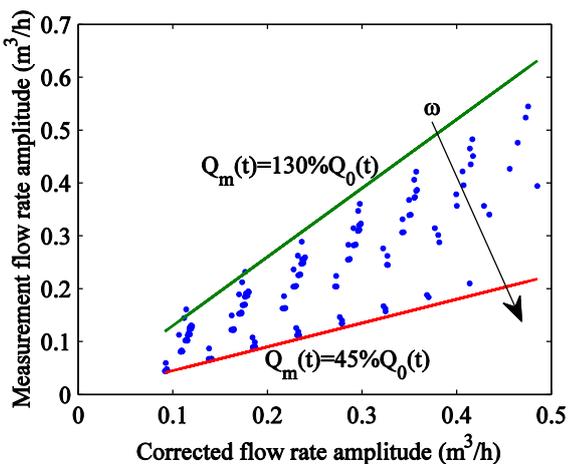


Fig. 8 Measured flow rate amplitude data vs. corrected flow rate amplitude data.

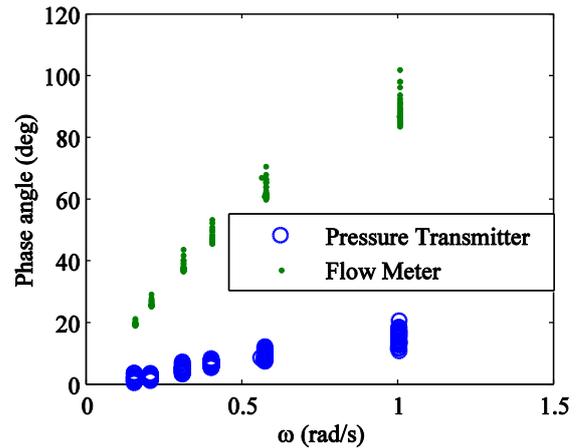


Fig. 9 The phase angle caused by sensor's dynamic response.

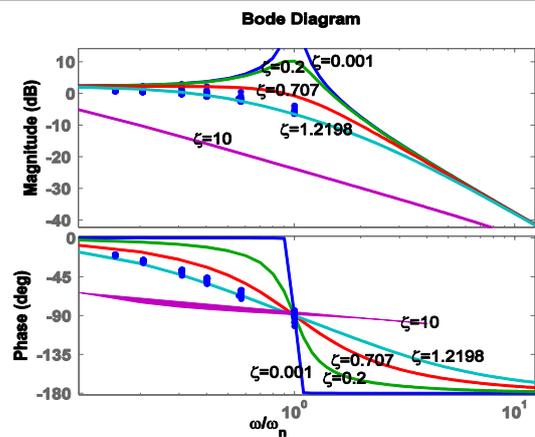


Fig.10 Bode diagram for the Flow Meter.

In Fig. 10, it gives some cases with different ζ . Firstly, as can be seen in Fig.10, the data by the authors of this paper fit the simplified Flow meter transfer function G_{Q_s} well. So it can be simplified as the mass-spring-damper model. Secondly, if the water's fluctuation frequency is very close to the system's natural frequency area, $\omega/\omega_n \approx 1$, both of the amplitude and the phase difference changes significantly with frequency, which is called resonance. While all of our experimental cases' frequency are close to this area, which is supposed to be the reason why the flow rate data vary from the fixed value so far. So, for the experiment or practical usage, we should avoid to measure the high frequency, which almost close to the sensor's natural frequency, otherwise the data need to be revised significantly.

6 Discussion

The authors of this paper would like to point out the following reservations as to the results obtained in 5.

(1) The method proposed in this paper is a kind of pre-treatment procedure to make the problem easier to handle by some simplification: The flow field is simplified as one-dimensional purely axial flow, with ignoring the radial flow component by negligible radial velocity and radial variation of pressure.

(2) The test section for this experiment is narrow rectangular channel. Since the authors of this paper did not take the specific resistance character, so this method is considered to be applicable for other shape's cross section. For the same reason it also may be applied to laminar, turbulence and two phase sinusoidal flow conditions. However, those issues have not tried in the authors' experimental study.

(3) DNS (Direct Numerical Simulation) method is reliable for calculating the flow condition in channel, which has no dynamic response measurement problem. So the authors' future work is to do some single phase and two phase sinusoidal flow simulation with DNS method to compare the simulation data with rivised experimental data to further understand the resistance pressure character.

6 Conclusion

In this paper, single-phase experiments were conducted in different sinusoidal flow condition for studying the dynamic response of measurement sensors. The obtained experimental data were fitted as the form of transfer function between flow rate signal as Input and pressure drop along the channel as Output of the dynamical system and to give the transfer function model of the second order model. By the derived transfer functions it becomes possible to trace the actual flow condition in the channel, by which it can be possible to measure magnitude deviation and the phase difference caused by the sensors' natural dynamic response delay.

Lastly, the following issues were also pointed out from the authors' conducted experimental studies.

(1) The pump with sinusoidal rotating speed will

give 2nd order Fourier series form integral pressure drop and friction pressure drop. It will force the fluid to sinusoidal form flow rate. The integral pressure drop and the flow rate naturally has phase difference which is caused by the acceleration of the water.

(2) The sensors' dynamic responding problem naturally exists, and it can cause measuring errors, acting as the sources of amplitude varying and phase difference.

(3) If the measurement object has the frequency which is accidentally close to the natural frequency of the sensor, it will cause significant errors, so that it should be avoided or modified. The dynamic system analysis theory with routine experiment provide an approach to calculate and correct this error.

Nomenclatures

A	Cross sectional area of experimental test section [m ²]
A_m	Amplitude
C_0	Average digital control voltage [V]
C_s	Digital control voltage amplitude [V]
G	Transfer function
G_l	Mass flow density [kg/m ² s]
G_P	Pressure drop's transfer function
G_{Pm}	Pressure Transmitter's transfer function
G_Q	Flow rate's transfer function
G_{Qm}	Flow Meter's transfer function
G_R	Pump's transfer function
I	Input signal
I_P	Pressure drop's input signal
I_{Pm}	Pressure Transmitter's input signal
I_Q	Flow rate's input signal
K_R	Coefficient of pressure to pump's rotating speed square
L	Distance of two pressures taps [m]
La	Laplace transform operator
M_R	Magnitude of pump's amplitude
P	Pressure [kPa]
O	Output signal
O_P	Output signal of Pressure Transmitter
O_Q	Output signal of Flow Meter

P_a	Acceleration pressure drop [kPa]
P_{aS}	Acceleration pressure drop amplitude [kPa]
P_0	Average pressure drop [kPa]
P_{f0}	Average friction pressure drop [kPa]
P_{fs}	Friction pressure drop amplitude [kPa]
P_S	Pressure drop amplitude [kPa]
P_{S2}	Pressure drop amplitude for the 2 nd order item [kPa]
Q	Flow rate [m ³ /s]
Q_0	Average flow rate [m ³ /s]
Q_S	Flow rate amplitude [m ³ /s]
Q_{mS}	Measurement value of flow rate amplitude [m ³ /s]
R	Pump's rotating speed [rad/s]
R_0	Average of pump's rotating speed [rad/s]
R_S	Amplitude of pump's rotating speed, [rad/s]
s	Laplace transform parameter, -
t	Time [s]
t_{Pm}	Pressure Transmitter delay time [s]
t_{Qm}	Flow Meter delay time[s]
T	period[s]
T_D	Digital control voltage period [s]
Y	Pump's control signal [V]
ρ	Water density [kg/m ³]
τ	Tme constant [-]
λ	Friction coefficient [-]
ω	Frequency [rad/s]
ω_n	Natural frequency [rad/s]
ζ	Damping ratio [-]
ψ	Phase angle [rad]
φ_a	Acceleration phase [rad]
φ_R	Pump's phase [rad]
φ_P	Pressure drop phase [rad]
φ_{Pm}	Measurement phase of Pressure Transmitter [rad]
φ_Q	Flow rate phase [rad]
φ_{Qm}	Measurement phase of Flow Meter [rad]

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References

- [1] SHAH, R.K., and LONDON, A.L.: Laminar flow forced convection in ducts. *Ada. Heat Transfer*, 1978
- [2] SADATOMI, Y., SATO, Y. and SARUVATARL, S.: Two-phase flow in vertical noncircular channels. *Int. J. Multiphase Flow*, 1982, 8(6): 641-655P
- [3] KONG, L.: *Engineering Fluid Mechanics*. M. Beijing, China Electric Power Press. 2001:116-117, 122-127
- [4] MELDA, Ö. Ç., and MEHMET, Y.G.: A critical review on pulsating pipe flow studies directing towards future research topics. *Flow Measurement and Instrumentation*, 2001, 12:163-17
- [5] GAO, P.Z., LIU, T.H., YANG, T., and TAN, S.C.: Pressure Drop Fluctuations in Periodically Fluctuation Pipe Flow. *J. Marine Sci. Appl.*, 2010, 9:317-312
- [6] BÉLA, G.: LIPTÁK (2003). *Instrument Engineers' Handbook: Process control and optimization* (4 ed.). CRC Press. p. 100. ISBN 0-8493-1081-4.
- [7] ADKINS, D.R. and BRENNEN, C.E.: Analysis of Hydrodynamic Radial Forces on Centrifugal Pump Impellers. *J. Journal of Fluids Engineering*, 1988(1): 20-28
- [8] SJÖBERG, J., ZHANG, Q., LJUNG, L., BENVENISTE, A., DEYLON, B., GLORENNEC, P., HJALMARSSON, H., and JUDITSKY, A.: "Nonlinear Black-Box Modeling in System Identification: a Unified Overview." *Automatica*. Vol. 31, Issue 12, 1995, pp. 1691–1724.
- [9] JUDITSKY, A., HJALMARSSON, H., BENVENISTE, A., DELYON, B., LJUNG, L., SJÖBERG, J., and ZHANG, Q.: "Nonlinear Black-Box Models in System Identification: Mathematical Foundations." *Automatica*. Vol. 31, Issue 12, 1995, pp. 1725–1750.
- [10] OPPENHEIM, J., and WILLSKY, A.S.: *Signals and Systems*. PTR Prentice Hall, Upper Saddle River, NJ, 1985.
- [11] DENNIS, J.E., JR., and SCHNABEL, R.B.: *Numerical Methods for Unconstrained Optimization and Nonlinear Eq.s*. PTR Prentice Hall, Upper Saddle River, NJ, 1983.
- [12] HASHEMIAN, H.M.: *Sensor Performance and Reliability [M]*. United State of America: The Instrumentation, System, and Automation Society. 2005, P105-P135
- [13] DEEPA, S. N., and SIVANANDAM, S. N.: *Control Systems Engineering, Using Matlab 2E*. India: Vikas Publishing House Pvt Ltd. 2009, P75