A new sensitivity measure of functional failure probability and a comprehensive sensitivity analysis procedure of thermal hydraulic passive systems

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Abstract: In this paper, a new sensitivity measure of functional failure probability to uncertain parameters of thermal hydraulic passive systems is proposed. The sensitivity measure is dependent on the partial derivative of functional failure probability with respect to distribution probability of uncertain parameters in inadmissible regions, in which uncertain parameters lead to greater functional failure probability than in other regions. This measure carries similar meaning as sensitivity of hardware failure probability to components failure probability. Thus, with this measure sensitivity of total failure probability of the system to uncertain parameters and components can be compared directly. A procedure for sensitivity analysis of total failure probability with respect to uncertain parameters and hardware components is presented. Steps in this procedure include 1) estimation of functional failure probability and sensitivity to uncertain parameters, 2) estimation of hardware failure probability and sensitivity to hardware components, and 3) calculation of total failure probability and sensitivity to these two types of factors, *i.e.* uncertain parameters and hardware components. A passive residual heat removal system is chosen for a case-study in feasibility validation of the proposed procedure. Sensitivity analysis is carried out and ranking of the uncertain parameters and components based on their contributions to the total failure probability is also presented.

Keyword: thermal hydraulic passive systems; functional failure; sensitivity measure; sensitivity analysis procedure

1 Introduction

A thermal hydraulic passive system may fail to deliver its intended functions due to stoppage of physical phenomena, such as stagnation of natural circulation flow in some heat exchanger tubes. This kind of failure mode is known as functional failure[1]. Nonzero probability of functional failure mode of thermal hydraulic passive systems has been covered in literatures [2-4]. Together functional and hardware failures contribute to total failure probability of passive systems. Functional failure can be caused by the deviation of uncertain parameters. For the passive system studied in this work, functional failure comes from uncertainties of thermal hydraulic parameters. Hardware failure is caused by failure of hardware components and is also investigated in this study. There are two types of factors influencing reliability of these systems, *i.e.* uncertain parameters and failure probability of hardware components. Comparison of sensitivity of total failure probability to these two types of factors is helpful in determining priorities of these factors in improving reliability of passive systems.

Various methods have been developed to estimate the functional failure probability of passive systems as well as sensitivity to uncertain parameters[4-7]. Among

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these methods, sensitivity of different forms, such as partial derivative of functional failure probability with respect to mean value and standard variation of uncertain parameters, are proposed. As for a hardware failure mode, some conventional methods, such as fault tree analysis, can be used to determine hardware failure probability and sensitivity to failure probability of hardware components. Sensitivity of functional failure probability, however, carries a different meaning as compared to that of the hardware failure probability. Within the previous investigation, sensitivities of total failure probability to uncertain parameters and hardware components are not compared directly.

In sensitivity analysis of hardware failure probability, the sensitivity indicator is defined as a product of two parts, *i.e.* the partial derivative of the hardware failure probability to the hardware components failure probability and the quotient of components failure probability divided by the hardware failure probability[8]. In accordance with the sensitivity indicator of hardware failure probability, the sensitivity of functional failure probability is also defined as a product of two parts.

In sensitivity analysis of functional failure probability, danger probability is defined as the probability of uncertain parameters located in the inadmissible regions, in which uncertain parameters may lead to higher functional failure probability than in other regions. With increase of danger probability of uncertain parameters, these parameters locate in the inadmissible regions with higher probability and deviate further from the expected values. In other words, the system performance can deviate further from the normal status. When the performance exceeds the allowable criterion, such as the lowest flow rate, the highest cladding temperature, deviation of uncertain parameters will result in a functional failure. For the sensitivity indicator, the first part is the partial derivative of the functional failure probability to the danger probability of uncertain parameters. The second part is the quotient of danger probability divided by the functional failure probability. This definition of sensitivity has the same meaning as criticality sensitivity of hardware failure probability to component failure probability [8, 9].

With the sensitivity defined, a procedure for comprehensive sensitivity analysis to uncertain parameters and hardware components is presented. This procedure enables sensitivities of the total failure probability to these two types of factors to be ranked together. Feasibility of the procedure is validated for a passive residual heat removal system (PRHR system). Sensitivity ranking of the total failure probability of the system to uncertain parameters and hardware components is presented.

This paper is organized as follows. Definition and calculation method of the new sensitivity measure are given in Section 2. The sensitivity analysis procedure is introduced in Section 3. A case study is described in Section 4, including description of the PRHR system, sensitivity calculation results, and corresponding analysis. Finally, conclusions are drawn in Section 5.

2 Sensitivity measure of functional failure probability

There are two requirements for the sensitivity measure proposed in this paper. First, it must reflect the influence of the uncertain parameters distribution on functional failure probability. Second, it should have the same mathematical meaning as the sensitivity of hardware failure probability.

The sensitivity of hardware failure probability is defined as partial derivative of hardware failure probability of the whole system with respect to the hardware failure probability of each component. Failure probability of a hardware component is regarded as the probability of component capacity exceeding the criterion, which is defined with the expected strength to support load. For a single uncertain parameter, a dangerous criterion is defined as the boundary of the inadmissible region. Danger probability of an uncertain parameter is the probability of the parameter to exceed the criterion.

A change in the danger probability leads to variation of the probability distribution of an uncertain parameter. Thus, distribution variation of output parameter of the system can occur, which causes variation of the failure probability in the system. The sensitivity measure proposed in this paper is to quantify in terms of partial derivatives of the functional failure probability of a passive system with respect to variations of danger probabilities of uncertain parameters.

Determination of dangerous criterion is the key to sensitivity measure. Considering that hardware failure sensitivity can also be affected by the component failure probability, danger probability of uncertain parameters is determined to be the same as the average value of component failure probability. Thus, influence of danger probability and failure probability on sensitivity comparison results is offset by the same value of the danger probability and average component failure probability. Furthermore, this sensitivity measure includes the quotient of danger probability divided by the functional failure probability, which also reduces the influence of danger probability on the sensitivity measure. As danger probability of an uncertain parameter is defined as the probability of the parameter exceeding the dangerous criterion, the dangerous criterion can be determined with the danger probability, distribution types and characteristic parameters.

The sensitivity of functional failure probability to danger probability of the uncertain parameter x_i is denoted by $S_{f\text{-}pi}$, and is defined as:

$$S_{f-pi}=\frac{\partial P_f}{\partial p_i}\times\frac{p_i}{P_f} \tag{1}$$

where P_f and p_i are functional failure probability of the system and danger probability of the uncertain parameter x_i respectively.

Eq. (1) can be represented by an expectation function:

$$\begin{aligned}S_{f-pi}&=\int\cdots\int_{\mathrm{F}}\frac{\partial f_X(x)}{\partial p_i}\times\frac{p_i}{P_f}\mathrm{d}x\\&=\int\cdots\int\frac{\partial f_X(x)}{\partial p_i}\times\frac{p_i}{f_X(x)}\times\frac{I_{\mathrm{F}}(x)f_X(x)}{P_f}\mathrm{d}x\\&=\mathrm{E}\left[\frac{\partial f_X(x)}{\partial p_i}\times\frac{p_i}{f_X(x)}\right]_{\mathrm{F}}\end{aligned} \tag{2}$$

where $X=(x_1, x_2, \ldots, x_n)$ is the n dimensional uncertain parameters vector, F is the failure region of the system in the space of uncertain parameters, $I_{\mathrm{F}}(x)$ is the indicator function of F, $I_{\mathrm{F}}(x)f_X(x)/P_f$ is the conditional probability density function (PDF) of X in failure region F.

For independent uncertain parameters, the joint PDF of X is a product of the PDF of x_i, that is, $f_X(x)=\prod_{i=1}^{n}f_{X_i}(x)$. Thus, the sensitivity of the uncertain parameter x_i can be represented with PDF of X_i, as the following:

$$S_{f-pi}=\mathrm{E}\left[\frac{\partial f_{X_i}(x)}{\partial p_i}\times\frac{p_i}{f_{X_i}(x)}\right]_{\mathrm{F}} \tag{3}$$

In this study, distributions of uncertain parameters are assumed to be truncated normal distribution with mean values corresponding to the nominal values. Due to lack of operational and experimental data, this probability distribution is determined by engineering/expert judgment [1, 10]. Then the PDF and danger probability of x_i can be represented as

$$f_{X_i}(x)=\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}}\Bigg/\int_{\min_i}^{\max_i}\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}}\mathrm{d}x \tag{4}$$

$$p_i=\int_{c_i}^{\max_i}\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}}\mathrm{d}x\Bigg/\int_{\min_i}^{\max_i}\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}}\mathrm{d}x \tag{5}$$

where μ_i, σ_i are the mean value and standard variation of x_i; $\max_i$, $\min_i$ are upper and lower limits of truncated distribution, and c_i is the dangerous criterion of x_i.

With this distribution, following results can be proved:

$$\mu_i<\frac{1}{2}(\max_i+c_i)\Rightarrow\frac{\partial p_i}{\partial\mu_i}>0 \tag{6}$$

$$\frac{\sigma_i}{\max_i-\mu_i}<\sqrt{\frac{k_i^2-1}{2\ln k_i}}\Rightarrow\frac{\partial p_i}{\partial\sigma_i}>0 \tag{7}$$

where $k_i=(c_i-\mu_i)/(\max_i-\mu_i)$.

When sufficient conditions of formulae (6) and (7) are satisfied, monotonicity of p_i with variation of μ_i, σ_i can be ensured. Then the partial derivative of the probability density function to the danger probability of uncertain parameters can be defined as:

$$\frac{\partial f_{X_i}(x)}{\partial p_i}=\frac{1}{2}\times\frac{\partial f_{X_i}(x)}{\partial\mu_i}\Bigg/\frac{\partial p_i}{\partial\mu_i}+\frac{1}{2}\times\frac{\partial f_{X_i}(x)}{\partial\sigma_i}\Bigg/\frac{\partial p_i}{\partial\sigma_i} \tag{8}$$

Furthermore, partial differential terms in the above equation can be developed as following, with the definition of PDF and danger probability:

$$\frac{\partial f_{X_i}(x)}{\partial \mu_i} \Big/ \frac{\partial p_i}{\partial \mu_i} = \frac{f_{X_i}(x)\times u_i(x)/\sigma_i - f_{X_i}(x)[f_{X_i}(\min_i)-f_{X_i}(\max_i)]}{\left[f_{X_i}(c_i)-f_{X_i}(\max_i)\right]-p_i[f_{X_i}(\min_i)-f_{X_i}(\max_i)]} \tag{9}$$

$$\frac{\partial f_{X_i}(x)}{\partial \sigma_i} \Big/ \frac{\partial p_i}{\partial \sigma_i} = \frac{f_{X_i}(x)\times\left[-1+u_i(x)^2\right]/\sigma_i - f_{X_i}(x)\left[f_{X_i}(\min_i)u_i(\min_i)-f_{X_i}(\max_i)u_i(\max_i)\right]}{\left[f_{X_i}(c_i)u_i(c_i)-f_{X_i}(\max_i)u_i(\max_i)\right]-p_i\left[f_{X_i}(\min_i)u_i(\min_i)-f_{X_i}(\max_i)u_i(\max_i)\right]} \tag{10}$$

where $u_i(x)=\dfrac{x-\mu_i}{\sigma_i}$.

$$S_{f-pi}=\mathrm{E}\left[\begin{array}{l}\dfrac{1}{2}\times\dfrac{u_i(x)/\sigma_i-[f_{X_i}(\min_i)-f_{X_i}(\max_i)]}{\left[f_{X_i}(c_i)-f_{X_i}(\max_i)\right]-p_i[f_{X_i}(\min_i)-f_{X_i}(\max_i)]}\times p_i+\\ \dfrac{1}{2}\times\dfrac{\left[-1+u_i(x)^2\right]/\sigma_i-\left[f_{X_i}(\min_i)u_i(\min_i)-f_{X_i}(\max_i)u_i(\max_i)\right]}{\left[f_{X_i}(c_i)u_i(c_i)-f_{X_i}(\max_i)u_i(\max_i)\right]-p_i[f_{X_i}(\min_i)u_i(\min_i)-f_{X_i}(\max_i)u_i(\max_i)]}\times p_i\end{array}\right]_{\mathrm{F}} \tag{11}$$

$$\hat{S}_{f-pi}=\frac{1}{N}\sum_{j=1}^{N}\left.\left[\begin{array}{l}\dfrac{1}{2}\times\dfrac{u_i(x_j)/\sigma_i-[f_{X_i}(\min_i)-f_{X_i}(\max_i)]}{\left[f_{X_i}(c_i)-f_{X_i}(\max_i)\right]-p_i[f_{X_i}(\min_i)-f_{X_i}(\max_i)]}\times p_i+\\ \dfrac{1}{2}\times\dfrac{\left[-1+u_i(x_j)^2\right]/\sigma_i-\left[f_{X_i}(\min_i)u_i(\min_i)-f_{X_i}(\max_i)u_i(\max_i)\right]}{\left[f_{X_i}(c_i)u_i(c_i)-f_{X_i}(\max_i)u_i(\max_i)\right]-p_i[f_{X_i}(\min_i)u_i(\min_i)-f_{X_i}(\max_i)u_i(\max_i)]}\times p_i\end{array}\right]\right|_{x_j\in \mathrm{F}} \tag{12}$$

S_{f-pi} can be represented as Eq. (11) with an algebraic equation containing the PDF, danger probability and other parameters, rather than a differential equation. In numerical calculation, the estimate of $S_{f\text{-}pi}$ can be calculated through Eq. (12) with distribution parameters μ_i, σ_i and sample points x_j in failure region.

Before sensitivity analysis, it should be examined that whether sufficient conditions of formulae (6) and (7) are satisfied.

3 Sensitivity analysis procedure

Sensitivity analysis of the passive system can be divided into two levels. At the first level, sensitivities of two failure probabilities to relevant influencing factors are analyzed. At the second level, sensitivity of the total failure probability to functional failure probability and hardware failure probability are calculated. Then, sensitivity of total failure probability to uncertain parameters and components is obtained based on the results from the first two levels of analysis.

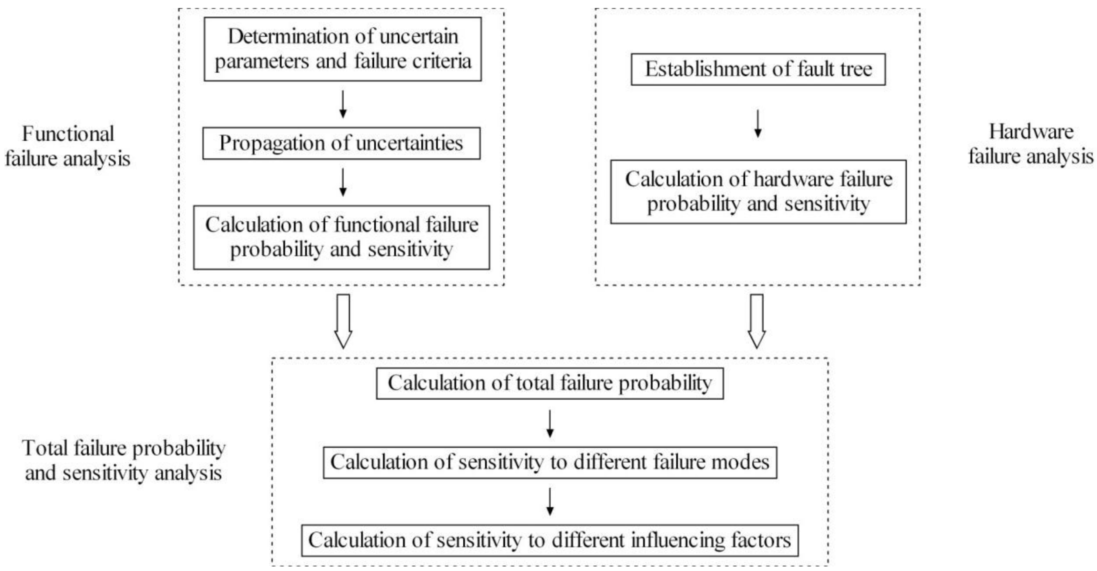

Fig. 1 Comprehensive sensitivity analysis procedure.

There are three major steps in the sensitivity analysis procedure, (1) functional failure analysis, (2) hardware failure analysis, and (3) total failure probability and sensitivity analysis. In the first step, the functional failure probability and its sensitivity to uncertain parameters are calculated with the measure proposed in Section 2. In the second step, the hardware failure probability and sensitivity to components are achieved with fault tree analysis method. As there is no dependency between these two steps, sequence of these two steps can be altered freely. In the third step, the total failure probability and sensitivity to uncertain parameters and components are obtained, as shown in Fig. 1.

3.1 Estimation of functional failure probability and sensitivity

Functional failure probability is determined by uncertainties and thermal hydraulic characteristics of the studied system which are described by a thermal hydraulic model. The basic idea of functional failure probability estimation is to identify uncertain parameters and to propagate uncertainties through the model to obtain probability of the system to reach failure criteria. As for sensitivity of the failure probability to uncertain parameters, it can be obtained with the measure proposed in Section 2 and sample points of uncertain parameters.

3.1.1 Determination of uncertain parameters and failure criteria

Uncertainties resulting in functional failure include model uncertainties and input uncertainties. Model uncertainties are uncertainties of expressions adopted in establishing a thermal hydraulic model of the system, such as expressions of heat transfer coefficients and friction resistance coefficients. This type of uncertainties can be captured by a multiplicative model [11-13]: $y=\mathrm{f}(x)\cdot\varepsilon$, where y is the real value of the variable to be calculated, $f(x)$ is the calculation result of the expression, and ε denotes uncertainty of the expression. Input uncertainties are input thermal hydraulic parameters, such as, power level, system pressure, and temperature of heat sink. Both these two types of uncertainties can be represented by subjective probability distributions [14].

Failure criterion of the system can be determined by defining a global failure criterion for the complete system, containing the safety system and the reactor core [5]. For safety systems used to cool the reactor core of nuclear power plants, failure criterion is always represented with a maximal temperature, such as temperature of the core outlet coolant, element cladding, and fuel pellet.

3.1.2 Propagation of uncertainty

In propagation from parameters uncertainties to functional failure probability, samples of uncertain parameters are drawn as input of thermal hydraulic program of the system and output are calculated. In practice, response surface functions are usually produced to replace the thermal hydraulic program in order to reduce computing time. Considering potential non-linearity of the system model, quadratic polynomials are selected as the form of response surface functions [2, 3, 5].

When the failure probability is low, a great number of samples are drawn to obtain results with high accuracy in Monte Carlo simulation. In this study, the Subset Simulation method is adopted in the functional failure probability estimation to improve efficiency of sampling. With the Subset Simulation method, conditional samples are drawn in a series of intermediate failure regions with inclusion relation. Thus the small failure probability is expressed as a product of several larger conditional failure probabilities. Comparing with Monte Carlo simulation method, simulation time can be significantly reduced with the Subset Simulation method. A detailed description of the method is available in [7, 15, 16].

3.1.3 Calculation of functional failure probability and sensitivity

Using the Subset Simulation method, functional failure probability is obtained. With sample points in the failure region and distribution characteristics of uncertain parameters, the estimate of sensitivity can be calculated as in Eq. (12). In derivation of this equation, danger probability is defined in the inadmissible region between upper limit of the parameter and dangerous criterion. In sensitivity calculation of the case studied, the partial derivative of functional failure probability to mean value of uncertain parameter is considered. When this partial derivative is positive, inadmissible region is defined in the region between the upper limit and dangerous criterion. As for the negative condition, inadmissible region is defined between the lower limit and dangerous criterion. In this way, inadmissible region defined always contain uncertain parameters leading to greater functional failure probability than parameters in other regions.

3.2 Estimation of hardware failure probability and sensitivity

Probability of hardware failure mode is determined by component failure probabilities and the system structure. In this study, the fault tree analysis (FTA) method [17] is applied to quantify hardware failure probability. Criticality importance [8, 9] is used as a sensitivity indicator of hardware failure mode. Because of linear relationship between failure probabilities of the whole system and each component, this indicator is defined and developed as:

$$\begin{aligned} S_{\mathrm{h}-ci} &= \frac{\partial P_{\mathrm{h}}}{\partial p_{ci}} \times \frac{p_{\mathrm{c}i}}{P_{\mathrm{h}}} \\ &= \left(P_{\mathrm{h}}\big|_{p_{\mathrm{c}i}=1} - P_{\mathrm{h}}\big|_{p_{\mathrm{c}i}=0} \right) \times \frac{p_{\mathrm{c}i}}{P_{\mathrm{h}}} \end{aligned} \tag{13}$$

where P_h and p_{ci} are hardware failure probability of the system and failure probability of the component i respectively.

3.3 Total failure probability and sensitivity analysis

Hardware failure is caused by failure of hardware components while functional failure is caused by deviation of some thermal-hydraulic parameters while hardware components keep intact well. Uncorrelated failure causes lead to independent relationship between functional failure mode and hardware failure mode. Thus total failure probability of the system P_{t} can be calculated through the following formula: with probabilities of these two failure modes, *i.e.* P_{f} and P_{h},

$$P_t = P_h + P_f - P_h \times P_f \tag{14}$$

Sensitivity of total failure probability to these failure modes can be obtained as:

$$S_{t-f} = \frac{\partial P_t}{\partial P_f} \times \frac{P_f}{P_t} = (1 - P_h) \times \frac{P_f}{P_h + P_f - P_h \cdot P_f} \tag{15}$$

$$S_{t-h} = \frac{\partial P_t}{\partial P_h} \times \frac{P_h}{P_t} = (1 - P_f) \times \frac{P_h}{P_h + P_f - P_h \cdot P_f} \tag{16}$$

Comprehensive sensitivity of total failure probability to two types of factors is obtained through two levels of sensitivity analysis:

$$\begin{aligned} S_{t-factor} &= \frac{\partial P_t}{\partial p_{factor}} \times \frac{p_{factor}}{P_t} \\ &= \frac{\partial P_t}{\partial p_{mode}} \times \frac{\partial p_{mode}}{\partial P_{factor}} \times \frac{p_{mode}}{P_t} \times \frac{P_{factor}}{p_{mode}} \\ &= S_{t-\mathrm{mod}e} \times S_{\mathrm{mod}e-factor} \end{aligned} \tag{17}$$

where: $S_{t-\mathrm{mod}e}$ is sensitivity of the total failure probability to a failure mode, including S_{t-f} for functional failure mode, S_{t-h} for hardware failure mode, and; $S_{\mathrm{mode}-factor}$ is sensitivity of the failure mode to relevant factor, including S_{f-pi} , S_{h-ci} . Definitions of these two parameters can be found in sections 2 and 3.2.

4 Case study

4.1 Description of the PRHR system

In this case study, the PRHR system in a pressurized water reactor (PWR) is considered to illustrate the sensitivity analysis. The purpose of this system is to remove core decay heat. Performance of the system following a station blackout accident is examined.

The schematic diagram of the system is shown in Fig.2. The key component of this system is a heat exchanger immersed in a water tank, which is located above the reactor core. Hardware of the system also contains a normal open valve, two redundant fail-open valves, a lock open valve, which is locked in open condition after it is opened, and relevant pipes connecting these components. This system is designed to maintain the safety shutdown condition of the reactor when an accident occurs. In the case of a station blackout accident, the coolant pumps in the primary loop do not work and feed water for the secondary side of the steam generators is unavailable. Core residual heat is removed by steam generators with water stored in secondary side. When the PRHR system is actuated, residual heat is removed mostly by this passive heat exchanger.

Failure of the system refers to failing to provide sufficient cooling capabilities to remove decay heat from the reactor core. Together, functional failure and hardware failure contribute to failure of the system. The former failure mode refers to failure of the system due to deviation of some uncertain parameters, such as increase of the flow resistance and decrease of heat transfer rate, which may lead to a significant increase of the core coolant outlet temperature. The latter one refers to failure due to loss of expected function or the rupture of hardware components, such as failing to open or a break in a valve, rupture of pipes and heat exchanger tubes.

4.2 Functional failure analysis

4.2.1 Uncertain parameters and failure criterion

In this study, eight tentative uncertainties are considered, including uncertain input parameters and model error factors, as listed in Table 1, which are estimated by the thermal hydraulic code PRHRSDSC [4]. The parameter ranges assign 95% confidence boundary of the nominal value. The standard deviation is obtained by $\sigma_i = |x_i - \mu_i| / Z_{0.95}$, where $Z_{0.95}$ is a distance of the left-side confidence level 0.95 of the standard normal distribution, and $Z_{0.95}$ =1.6449 [10]. A core coolant outlet temperature of 350℃ is considered as failure criterion of the passive system [18]. When core coolant outlet temperature exceeds this criterion, occurrence of heat transfer deterioration between the fuel pin and coolant is expected, which can result in boiling and flow instability in the primary coolant system [4, 19].

4.2.2 Propagation of uncertainties

A response surface function of core coolant outlet temperature and uncertain parameters is used to replace the thermal hydraulic program. With calculation results of the program and uncertain parameters, the response surface function of these eight parameters is constructed through least squares method:

$$\begin{aligned}\hat{y}(X) = {} & 141.616+6.65\text{e-}06Q_{\mathrm{u}}+0.9T_{\mathrm{CT}}+0.12P \\ & +12.54k_{\mathrm{loc}}-34.23\xi_1-193\xi_2+4.25\xi_3-5.01\xi_5 \\ & -2.34\text{e-}14Q_{\mathrm{u}}-3.10\text{e-}04T_{\mathrm{CT}}-6.97\text{e-}04P \\ & -2.13k_{\mathrm{loc}}+11.37\xi_1+6.07\xi_2-0.38\xi_3+5.21\xi_5\end{aligned} \tag{18}$$

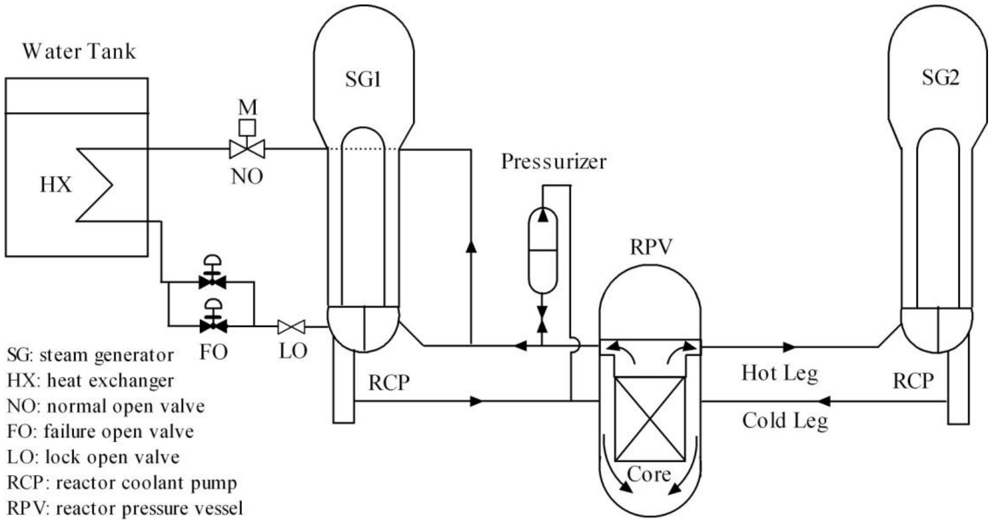

Fig. 2 Schematic diagram of the PRHR system

Table 1 Distribution characteristics of uncertain parameters

	Parameters	Distribution	μ	σ	Range	Remarks
Uncertainties of input parameters	Q_{u}/MW	normal	220	3.10	(214.9, 225.1)	Reactor decay heat
	T_{CT}/℃	normal	50.0	4.46	(43.8, 56.2)	Fluid temperature of the water tank
	P/MPa	normal	15.5	0.67	(14.4, 16.6)	Pressure of the coolant loop
	k_{loc}/(kg·m)$^{-1}$	normal	1.7	0.16	(1.44, 1.96)	Pressure drop coefficients
Uncertainties of models (error factor, ζ)	ζ_1	normal	1.0	0.06	(0.9, 1.1)	Heat transfer coefficient in forced convection single-phase regime
	ζ_2	normal	1.0	0.15	(0.75, 1.25)	Heat transfer coefficient in pool boiling regime
	ζ_3	normal	1.0	0.12	(0.8, 1.2)	Heat transfer coefficient in forced convection boiling regime
	ζ_5	normal	1.0	0.03	(0.95, 1.05)	Friction factor in single-phase flow regime

In the existing application of Subset Simulation method, Markov Chain is adopted to produce conditional samples with samples of the prior failure region[7]. With this method, however, different selections of proposal distribution of Markov Chain may result in different probability estimation outcomes. In this study, the Subset Simulation method based on Genetic Algorithm (SSGA) is proposed. In this method, the Genetic Algorithm is used for producing conditional samples. Sample points with largest performance function values in prior failure region are used as parental points. Pre-candidate samples are generated after hybridization and mutation. If values of the performance function and the conditional probability density function of a sample are greater than those of the parental points, this sample is selected as a conditional sample of the next failure region.

In implementation of the SSGA method, the conditional failure probability of prior failure region is determined to be 0.1 according to the method suggested in [20]. Results show that 32 out of 100 sample points exist in the third level of failure region with output exceeding failure criterion. Functional failure probability of the system can be obtained by multiplying conditional failure probabilities of all levels of failure regions, which are 0.1, 0.1 and 0.32.

4.2.3 Functional failure probability and sensitivity

With the SSGA method, functional failure probability of the system is calculated to be $P_f = 3.2\times10^{-3}$. The failure probability is also examined by direct Monte Carlo (DMC) method, and the result of this analysis is 3.223×10^{-3}. Relative error of the result of SSGA method is 0.7136%. But time cost with this method is much less than that of DMC method. Estimation of functional failure probability in this paper provides a basis for comprehensive sensitivity analysis. The result of functional failure probability can affect accuracy of the comprehensive sensitivity analysis results, but it has no influence on feasibility validation of the comprehensive sensitivity analysis procedure proposed in this paper.

With danger probability of uncertain parameters, which are determined to be the same as the average value of component failure probability, danger criteria are obtained. Then sufficient conditions of formulae (6) and (7) are examined. Calculation results show that these sufficient conditions are satisfied.

With sample points, sensitivity of functional failure probability to danger probability of uncertain parameters is obtained, as shown in Fig. 3. This result is compared with that in literature [4], in which the system studied is the same to that in this paper. Four of eight uncertain parameters with greater sensitivity are k_{loc}, Q_{u}, T_{CT} and ζ_2 in this paper, while in literature [4], four of these eight uncertain parameters with greater sensitivity are k_{loc}, Q_{u}, T_{CT} and ζ_3 when two sensitivity sequences obtained with mean sensitivity and standard deviation sensitivity are considered. This difference lies in difference of analysis method. Validation of the sensitivity analysis results can be carried out by examining change of functional failure probability resulting from change of uncertain parameters distributions. This work will be carried out in the future.

4.3 Hardware failure analysis

In hardware failure analysis, a fault tree is built according to the system structure. Hardware failure of the system is considered as the top event of this fault tree and failure of individual components are regarded as bottom events. Besides failure of devices, failures of control signals of the normal open valve and fail-open valves are also taken into account, as depicted in Fig. 4.

With the failure probability data suggested in [21], the hardware failure probability of the system is calculated to be $P_h=3.01\times10^{-4}$. With critical importance as an indicator, sensitivity of hardware failure probability to components is obtained, as illustrated in Fig. 5. It is observed that sensitivities to redundant components are less than other components.

4.4 Total failure probability and sensitivity analysis

Based on analysis of these two failure modes, the total failure probability of the system is calculated to be $P_t=3.5\times10^{-3}$. Sensitivities to functional failure mode and hardware failure mode are: S_{t-f} =0.914, S_{t-h} =0.0857, respectively. The fact that sensitivity of the total failure probability to functional failure is higher than to hardware failure results from that the functional failure probability is obviously greater than the hardware failure probability. The sensitivity to uncertain parameters and hardware components are calculated with results of these two levels of sensitivity analysis, as shown in Fig. 6. It can be concluded that sensitivities to Q_u, T_{CT}, ζ_2, k_{loc}, ζ_1, ζ_3, lock open valve (LO V), normal open valve (NO V), heat exchanger (HX) and ζ_5 are greater than other factors, while sensitivities to these redundant hardware components are less. These six factors with the largest sensitivity are uncertain parameters influencing functional failure. This result is partly attributed to the higher sensitivity of the total failure probability to functional failure mode than to hardware failure mode.

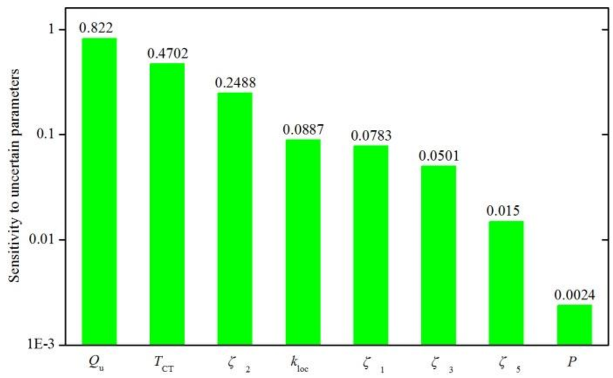

Fig. 3 Sensitivity of functional failure probability to uncertain parameters.

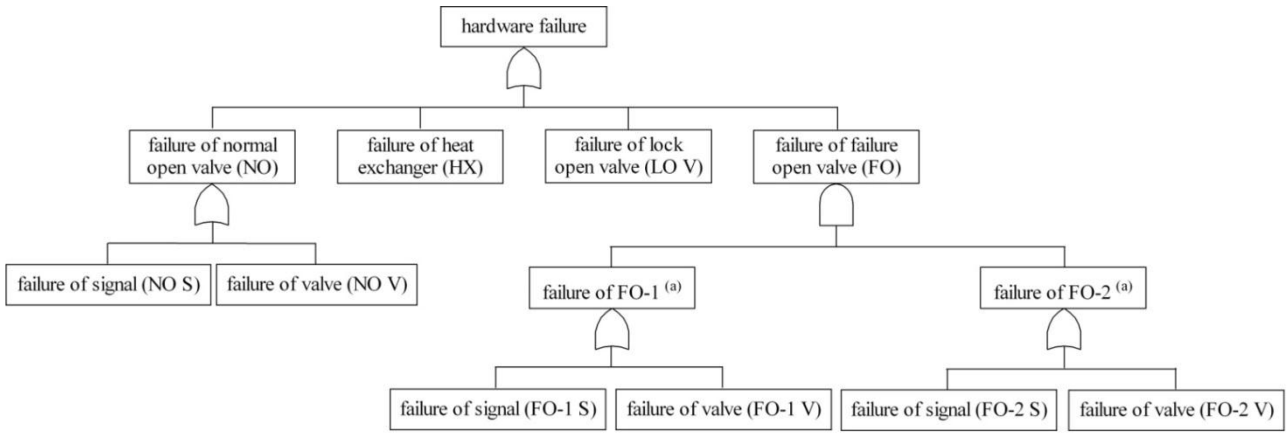

Fig. 4 Fault tree of hardware failure mode analysis

(a) FO-1 and FO-2 denote two redundant failure open valves.

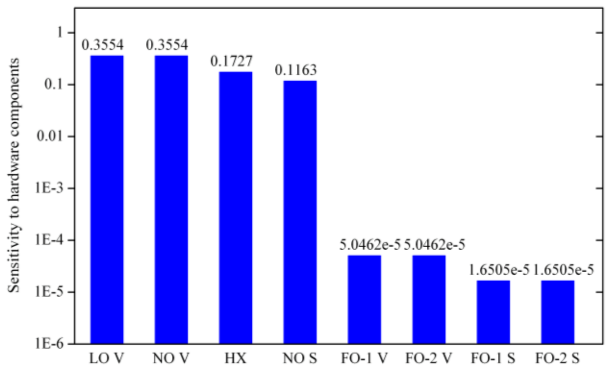

Fig. 5 Sensitivity of hardware failure probability to components.

5 Conclusions

In this paper, a new sensitivity measure of functional failure probability to uncertain parameters is proposed. This measure has similar meaning as that of hardware failure. Based on this measure, a comprehensive sensitivity analysis procedure is developed. This procedure can be used to analyze sensitivity to uncertain parameters and hardware components. A passive system within a PWR is considered as a case-study. Total failure probability and sensitivity to uncertain parameters and hardware components are calculated. Results show that, with the proposed measure and procedure, sensitivity ranking of different types of failure factors can be obtained. This ranking is helpful to determine priorities of uncertain parameters and hardware components for reliability improvement within the system. Specially, when the total failure probability is sensitive to functional failure and relevant uncertain parameters, as in the case of the current study, measures can be taken to reduce parameters uncertainties improved system design, construction, and increased operational margin.

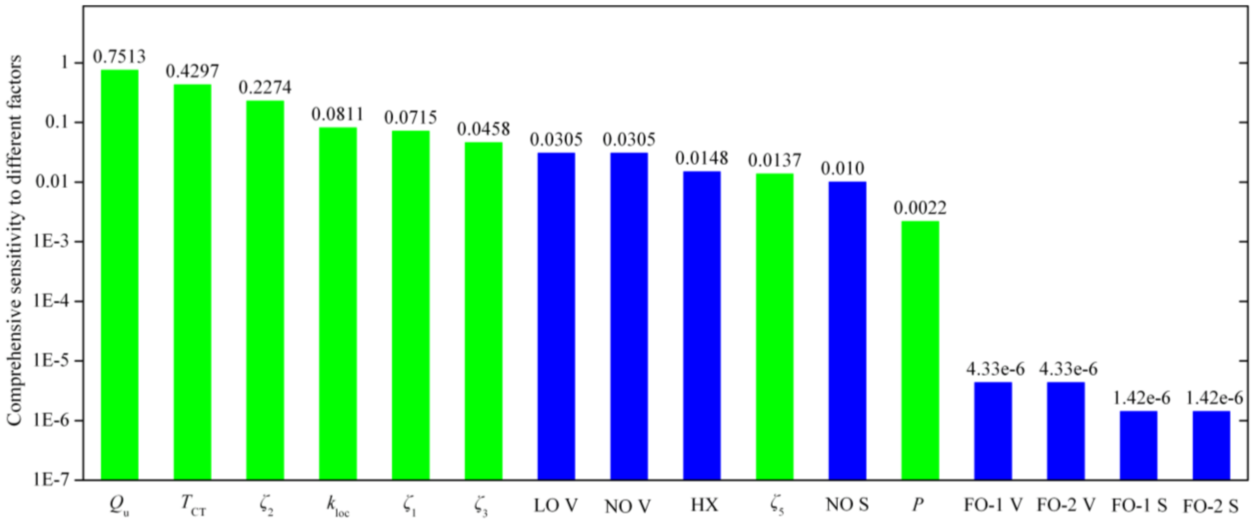

Fig. 6 Sensitivity of total failure probability to uncertain parameters and hardware components.

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