Modeling of natural circulation system with and without reactivity feedback

CHEN Xianbing, and GAO Puzhen[1]

1. Fundamental Science on Nuclear Safety and Simulation Technology Laboratory, Harbin Engineering University, Harbin, 150001, PR China (gaopuzhen@hrbeu.edu.cn)

Abstract: A nuclear power simulation system is model with reactivity feedback taking into consideration. The power feedback algorithm is developed by representing reactor dynamic with a point reactor model. Homogeneous flow model is used to predict thermal-hydraulic behavior in the natural circulation loop. The governing equations of mass, momentum and energy coupled with point reactor model are discretized and solved. Simulation of natural circulation system is carried out with and without reactivity feedback. Numerical results on flow instability with different amplitudes and periods are found and compared with experimental data. The transient behavior of power step is analyzed under reactivity feedback mode. Reactivity feedback is found to have a significant influence on natural circulation transient.

Keyword: natural circulation; reactivity feedback; thermal-hydraulic; point reactor model; flow instability

1 Introduction

Next generation reactors adopt natural circulation operation to gain passive safety features. Compared with forced circulation mode, natural circulation is driven by the buoyance force resulting from density difference of the working fluid. For light water reactors, water is used not only as coolant but also moderator. Change of water temperature and void fraction would cause disturbance for reactivity. Besides, fuel temperature can also bring thermal Doppler broadening and result in neutron density change. Reactivity feedback is supposed be taken into consideration for the modeling of natural circulation transient behavior.

The modeling of thermal-hydraulic characteristics has been reported by many researchers. Guo *et al.* used a homogeneous flow model to establish the stability map for a research reactor [1]. Jain and Rizwan also developed a computer code based on homogeneous flow model to investigate the supercritical flow instability [2]. Drift flux model has been widely used for the prediction of two-phase flow boiling phenomenon due to its consideration of relative motion between two phases [3-6]. Hibiki and Ishii derived the constitutive equations for drift flux model[7]. Drift flux model has been used to study the stability behavior of natural circulation [8, 9], LOCA analysis [10-12] and small reactors [13]. In addition, effects of reactivity feedback on the stability of natural circulation system have been experimentally and numerically investigated. Rao et al. proposed an experimental simulator which is capable of computing void-reactivity feedback. Stability map was given on the axes of void-reactivity coefficient and fuel time constant [14]. Furuya et al. designed a void-reactivity feedback simulation loop to study the regional and core-wide stability. The match of fuel rod time constant and oscillation period is found to have a significant influence on regional instability [15]. Shi *et al.* tested the effects of void-reactivity on startup transients [16]. T'Joen and Rohde found different results on stability characteristics: a clear instability zone can be found under coupled neutronic-thermal-hydraulic mode, but no instability happens under thermos-hydraulic mode [17]. Lee and Pan presented nonlinear analysis about Type-I and Type-II instability. Void-reactivity feedback has destabilizing effects on both instabilities [18]. Wang *et al.* pointed out that increasing void-reactivity feedback coefficient has a destabilizing effect. On the other hand, increasing the Dopper effect feedback effect coefficient has a stabilizing effect [19]. Differently, Durga and Pandey indicated that increase in the absolute value of void reactivity coefficient has a stabilizing effect in Type-I region and destabilizing effect in Type-II region. The

Received date: September 3, 2018
(Revised date: January 10, 2019)

fuel time constant has a stabilizing effect on Type-II region and destabilizing effect on Type-I region [20].

The effects of reactivity feedback on natural circulation depend on the system geometry, working conditions and fuel type. Modeling of transient behavior of natural circulation system with reactivity feedback is essentially necessary for the design and safety analysis of reactors.

2 Power feedback

An experimental loop is designed to investigate the effects of nuclear power feedback on the transient behavior of natural circulation system. The schematic diagram of the experimental loop is shown in Fig.1. Water is used as the working fluid. The stainless steel test section is uniform heated by a DC power supply. A tube-shell shape condenser is mounted at the top. Heat generated in the test section is taken away in the condenser by the coolant water which flows in the secondary loop. System pressure is sustained by a Nitrogen tank pressurizer.

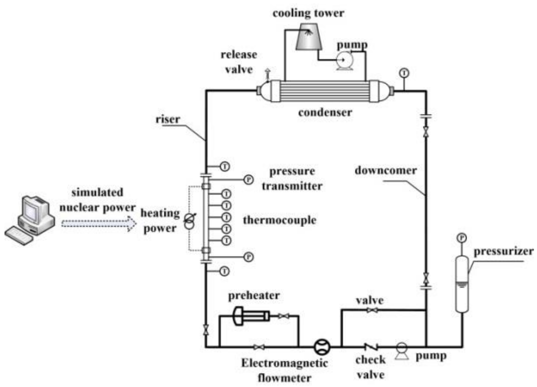

Fig.1 Experimental loop and power feedback.

Nuclear power is simulated by the point reactor kinetics model and represented by thermal power applied to the test section. The feedback loop is shown in Fig.2.

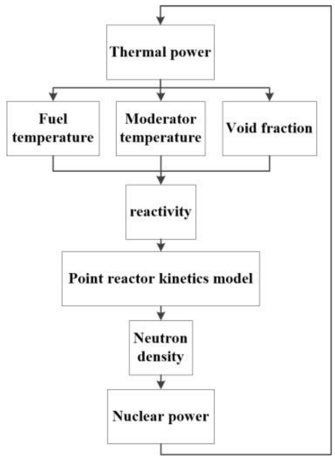

Fig.2 Power feedback algorithm.

3 Analytical model

Subcooled water is uniformly heated in the test section. Single-phase and two-phase flow are supposed to be carefully taken care of. Heat loss to the ambient from riser and down-comer are ignored by surrounding these tubes with thermal insulation layer. Flow boiling along the test section is analyzed by a 1-D axial flow model. Following assumptions are adopted in the model: (1) A 1-D homogeneous flow model is used. Generally the exit quality is quite low ($x_e \approx 0$) when flow instability condition is reached, the slip between single-phase liquid and two-phase vapor can be ignored as a result. (2) The inlet water temperature and system pressure are constant. (3) Heat flux along the test section is uniform. (4) Subcooled boiling is neglected.

3.1 Conservation equations

Mass conservation equation:

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}=0 \tag{1}$$

Momentum conservation equation:

$$\frac{\partial \rho u}{\partial t}+\frac{\partial \rho u^2}{\partial x}=-\frac{\partial P}{\partial x}-\rho g-F \tag{2}$$

Energy conservation equation:

$$\frac{\partial \rho h}{\partial t}+\frac{\partial \rho u h}{\partial x}=q_v \tag{3}$$

A state equation is needed to solve the mixture density:

$$\rho = eos(P,h) \tag{4}$$

3.2 Conservation equations

The heating wall has a thickness of 1 mm and the therefore heat capacity of the heated channel is ignored. Heating wall energy equation is given in equation (5). The RHS of equation (5) is the net volumetric heat release rate which is the difference between the electric heat generated by Jour heat q_{heater} and heat absorbed by the fluid q_v. The relationship between volumetric heat release rate and heat flux per unit area is given in equation (7).

$$\rho_w C_{pw} \frac{\mathrm{d}T_w}{\mathrm{d}t} = q_{net} \tag{5}$$

Net volumetric heat release rate:

$$q_{net} = q_{heater} - \frac{4D}{D_o^2 - D_i^2} q_s \tag{6}$$

Volumetric heat absorbed by the fluid:

$$q_v = \frac{4D_i}{D_o^2 - D_i^2} q_s \tag{7}$$

Heat flux from heating surface to fluid:

$$q_s = \bar{h}(T_w - T) \tag{8}$$

3.3 Closure correlations

The third term on RHS of equation (2) F is the frictional pressure drop. Single-phase flow frictional pressure drop can be obtained from:

$$\frac{\mathrm{d}P_f}{\mathrm{d}z} = \frac{\lambda}{2D} \rho u^2 \tag{9}$$

Two-phase flow frictional pressure drop is calculated based on a two-phase multiplier approach:

$$\frac{\mathrm{d}P_f}{\mathrm{d}z} = \phi_l^2 (\frac{\mathrm{d}P_f}{\mathrm{d}z})_l = \phi_g^2 (\frac{\mathrm{d}P_f}{\mathrm{d}z})_g \tag{10}$$

Heat transfer coefficients are calculated from:

$$Nu = \frac{\bar{h}d}{k_f} \tag{11}$$

Two semi-empirical correlations are chosen to calculate heat transfer coefficient in single-phase region and two-phase flow region. Single-phase flow boiling heat transfer coefficient is calculated using Dittus-Boelter correlation [21]:

$$Nu = 0.023 \times \mathrm{Re}^{0.8} \times \mathrm{Pr}^{0.4} \tag{12}$$

Liu-Winterton correlation [22] is chosen for two-phase heat transfer:

$$h_{TP}^2 = (Fh_L)^2 + (Sh_{pool})^2 \tag{13}$$

$$F = \left[1 + xPr_l(\frac{\rho_f}{\rho_g} - 1)\right]^{0.35} \tag{14}$$

$$S = \left(1 + 0.055 F^{0.1} Re_L^{0.16}\right)^{-1} \tag{15}$$

$$h_L = 0.023(k_l / d) Re_L^{0.8} Pr_l \tag{16}$$

$$h_{pool} = 55 p_r^{0.12} q^{2/3} (-\log_{10} p_r)^{-0.55} M^{-0.5} \tag{17}$$

Void fraction is calculated based on slip velocity ratio model. Slip ratio are calculated from Bankoff correlation [23]:

$$\alpha = K \Big/ 1 + (\frac{1 - x_e}{x_e}) \frac{\rho_g}{\rho_f} \tag{18}$$

$$K = 0.71 + 1.45 \times 10^{-8} P \tag{19}$$

3.4 Numerical approach

The governing equations are discretized on the staggered grid mesh arrangement as shown in Fig.3. The discrete values for velocity are stored on the cell faces and other variables (density, pressure and enthalpy) are stored at the cell center. The first-order upwind scheme is used to discretize the convection term in the mass, momentum and energy equations.

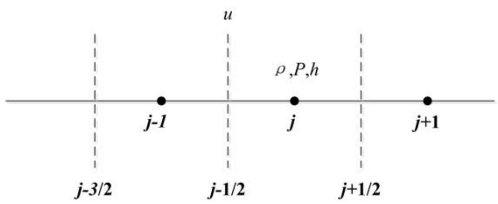

Fig.3 staggered grid mesh arrangement.

The discretized mass, momentum and energy equation are as listed in equation (20) to equation (22). There are four unknown variables (ρ,u P and h) to be solved at each time step for each grid point. These four variables are given in equation (23) to equation (26).

The Heating wall energy equation is discretized as:

$$(T_w)_j^{n+1} = (T_w)_j^n + \frac{(q_{net})_j^{n+1} \Delta t}{\rho_w C_{pw}} \tag{27}$$

The solution algorithm is given in Fig.4.

The discretization of mass conservation equation:

$$\frac{\rho_j^{n+1} - \rho_j^n}{\Delta t} + \frac{\rho_j^{n+1} u_{j+\frac{1}{2}}^{n+1} - \rho_{j-1}^{n+1} u_{j-\frac{1}{2}}^{n+1}}{\Delta x} = 0 \tag{20}$$

The discretization of momentum conservation equation:

$$\frac{(\rho u)_j^{n+1}-(\rho u)_j^n}{\Delta t}+\frac{\rho_j^{n+1}(u_{j+\frac{1}{2}}^{n+1})^2-\rho_{j-1}^{n+1}(u_{j-\frac{1}{2}}^{n+1})^2}{\Delta x}+\frac{P_j^{n+1}-P_{j-1}^{n+1}}{\Delta x}+\rho_j^{n+1}g+F_j^{n+1}=0 \tag{21}$$

The discretization of energy conservation equation:

$$\frac{\rho_j^{n+1}h_j^{n+1}-\rho_j^n h_j^n}{\Delta t}+\frac{\rho_j^{n+1}h_j^{n+1}u_{j+\frac{1}{2}}^{n+1}-\rho_{j-1}^{n+1}h_{j-1}^{n+1}u_{j-\frac{1}{2}}^{n+1}}{\Delta x}=(q_v)_j^{n+1} \tag{22}$$

Velocity:

$$u_{j+\frac{1}{2}}^{n+1}=\frac{\rho_{j-1}^{n+1}u_{j-\frac{1}{2}}^{n+1}-\frac{\Delta x}{\Delta t}(\rho_j^{n+1}-\rho_j^n)}{\rho_j^{n+1}} \tag{23}$$

Pressure:

$$\begin{aligned}P_j^{n+1}=P_{j-1}^{n+1}-\frac{\Delta x}{\Delta t}\left[(\rho u)_j^{n+1}-(\rho u)_j^n\right]-\left[\rho_j^{n+1}(u_{j+\frac{1}{2}}^{n+1})^2-\rho_{j-1}^{n+1}(u_{j-\frac{1}{2}}^{n+1})^2\right]\\ -\rho_j^{n+1}g\Delta x-\frac{\lambda_j^{n+1}}{2d}\rho_j^{n+1}\left(\frac{u_{j+\frac{1}{2}}^{n+1}+u_{j-\frac{1}{2}}^{n+1}}{2}\right)\Delta x\end{aligned} \tag{24}$$

Enthalpy:

$$h_j^{n+1}=\frac{(q_v)_j^{n+1}\Delta x+\frac{\Delta x}{\Delta t}\rho_j^n h_j^n+\rho_{j-1}^{n+1}h_{j-1}^{n+1}u_{j-\frac{1}{2}}^{n+1}}{\frac{\Delta x}{\Delta t}\rho_j^{n+1}+\rho_j^{n+1}u_{j+\frac{1}{2}}^{n+1}} \tag{25}$$

Density:

$$\rho_j^{n+1}=eos(P_j^{n+1},h_j^{n+1}) \tag{26}$$

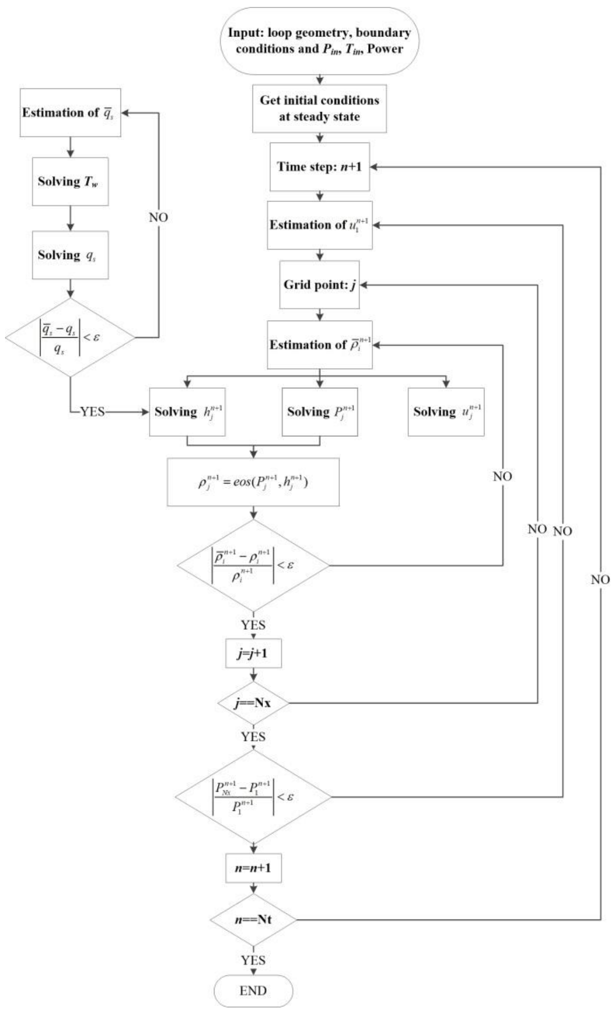

Fig.4 Code flow chart.

3.5 Point reactor kinetics

In the absence of extraneous neutron resource, point kinetics equations with six delayed neutron groups are solved by the fourth-order Runge-Kutta method to determine the fission power. The dynamic equations of neutron density and delayed neutrons are expressed in equation (28) and equation (29).

$$\frac{dn}{dt}=\frac{\rho-\beta}{l}n+\sum_{i=1}^{6}\lambda_i c_i \tag{28}$$

$$\frac{dc_i}{dt}=\frac{\beta_i}{l}n-\lambda_i c_i (i=1,2,...6) \tag{29}$$

The reactivity is a function of fuel temperature, moderator temperature (For single-phase working conditions, void reactivity feedback can be neglected).

$$\rho=\rho_0+\kappa_{Tw}\left(T_w\text{-}T_{w0}\right)+\kappa_{Tf}\left(T_f\text{-}T_{f0}\right) \tag{30}$$

Table 1 Neutronic data for point reactor model

Delayed neutrons fraction	Delayed neutrons decay constant
β_1=0.000266	λ_1=0.0127s^{-1}
β_2=0.001419	λ_2=0.0317s^{-1}
β_3=0.001316	λ_3=0.115s^{-1}
β_4=0.002849	λ_4=0.311s^{-1}
β_5=0.000896	λ_5=1.40s^{-1}
β_6=0.000182	λ_6=3.87s^{-1}
β=0.007	

The neutronic data for point reactor model with six groups of delayed neutrons are listed in Table 1. Neutron generation time is 0.00002s.

4 Result

4.1 Natural circulation without reactivity feedback

The heating power is kept constant under no reactivity feedback mode. The transient response of mass flux after a small perturbation is shown in Fig.5. For the stable working conditions, the mass flux before and after perturbation can be kept stable. For the instability conditions, mass flux undergoes self-sustained oscillations.

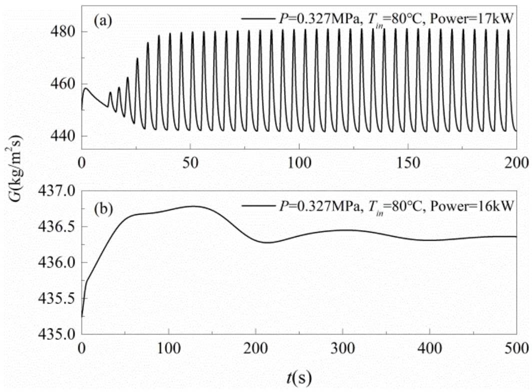

Fig.5 Transient response of mass flux.

Three typical flow oscillations characterized by different amplitudes and periods are found (as shown in Fig.6). At relative low power level, the flow oscillations are regular type with constant amplitude and period as shown in Fig.6 (a). However, with the increase of heating power, flow oscillations present as multiple amplitudes and periods as shown in Fig.6 (b). The natural circulation flow turn chaotic when heating power is further increased as shown in Fig.6 (c).

The calculated results are compared with experimental data on the Marginal Stability Boundary map as shown in Fig.7. Generally phase change number (equation (31)) and subcooling number (equation (32)) are employed to present the MSB. The predicted instability boundary indicated by dashed line lies close to x_e=0.

$$N_{sub} = \frac{h_f - h_{in}}{h_{fg}} \cdot \frac{\rho_f - \rho_g}{\rho_g} \tag{31}$$

$$N_{pch} = \frac{q}{Gh_{fg}} \frac{\rho_f - \rho_g}{\rho_g} \tag{32}$$

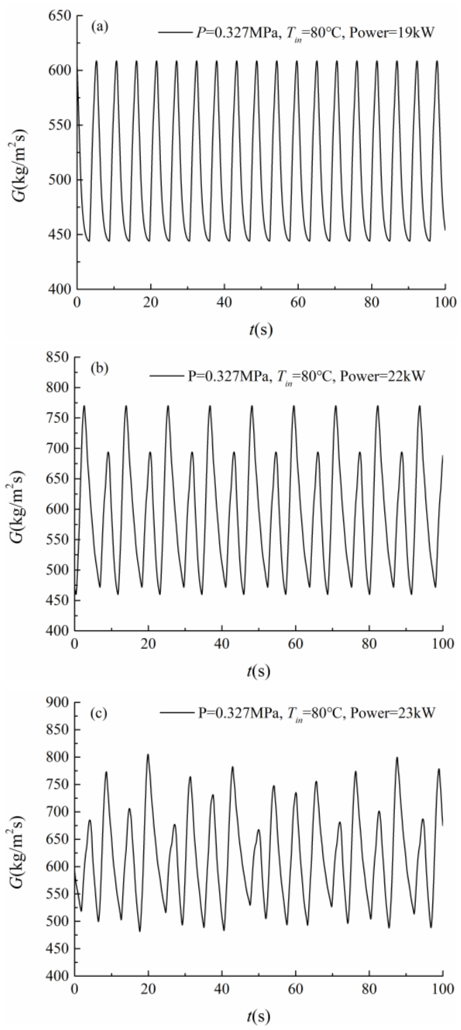

Fig.6 Flow instability.

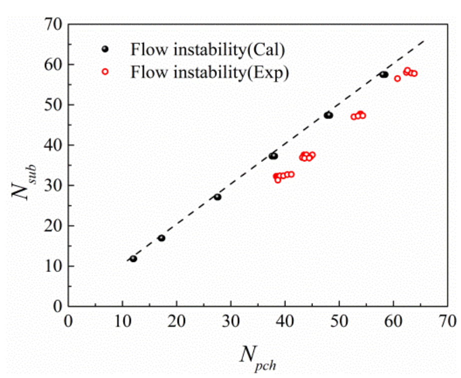

Fig.7 Stability map.

4.2 Natural circulation with reactivity feedback

A power step transient is simulated with and without reactivity feedback. Fluid inside the test section is single-phase flow so the contribution of void-reactivity feedback can be ignored. Effects of moderator temperature and fuel temperature are taken into consideration. The response of mass flux under power step from 10**kW** to 15**kW** is shown in Fig.8 by the black solid line. The transition from stable condition under 10**kW** and 15**kW** are obtained. Mass flux does not increase to the high value when reactivity feedback is introduced. Mass flux gets back to a same stable level after enough time. Increasing the feedback coefficient can decrease the peak value for mass flux during transition.

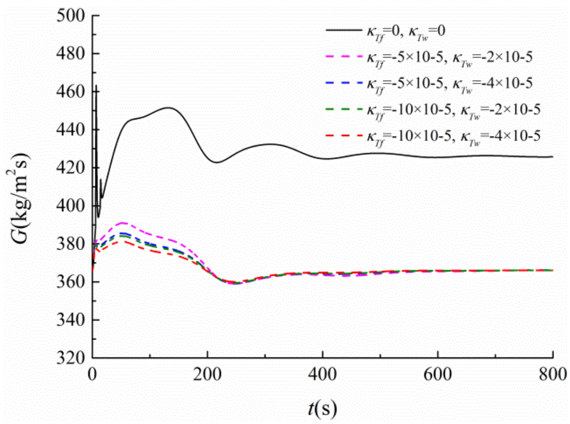

Fig.8 Mass flux.

The introduction of reactivity feedback has a great influence on the power step transition process as shown in Fig.9. Water temperature and fuel temperature increases when power is increased to a high value. Reactivity decreases after temperatures are increased to the highest value due to the negative feedback effect. Power decreases to the initial value after enough time as a result. It can be seen from this simple case that negative reactivity feedback benefits the neutronic-thermal-hydraulic system under power step transient.

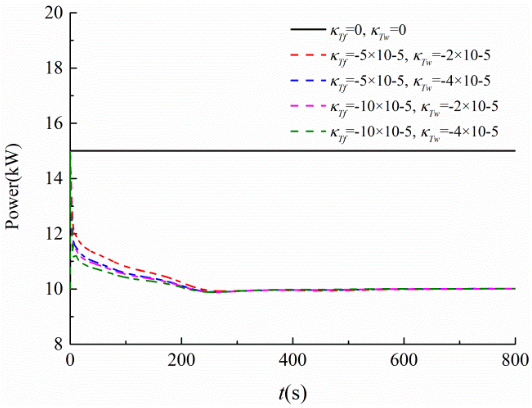

Fig.9 Power feedback.

5 Conclusion

A power feedback algorithm is developed to simulate nuclear heat release in an experimental facility in this paper. The modeling of nuclear power simulation system is carried out. Transient behavior of natural circulation system under small perturbation is investigated. Numerical results about flow instability are obtained and compared with experimental data. The effects of reactivity feedback are studied by a power step transient.

A natural circulation flow model is developed with reactivity feedback taking into consideration. The thermal-hydraulic flow model coupled with a point reactor model is numerically solved. Three typical types of flow instabilities with different amplitudes and periods are found under different heating powers. Natural circulation flow may become chaotic at high heating power. The numerical results are compared with experimental data. Satisfactory agreement is achieved. In addition, transient responses of the natural circulation system under power step with and without reactivity feedback are obtained. The introduction of reactivity feedback has a significant influence on natural circulation flow and power. Negative reactivity feedback benefits the neutronic-thermal-hydraulic system under power step transient.

Nomenclature

c	Delayed neutron precursor concentration(1/m^3)
C_p	Specific heat(kJ/°C)
D	Diameter(m)
F	Frictional pressure drop(kPa) /Enhanced factor
g	Gravitational acceleration(ms^{-2})
G	Mass flux(kg/m^{2}s)
h	Enthalpy(kJ/kg)
$\overline{h}$	Heat transfer coefficient(kJ/m^2°C)
k	Thermal conductivity(kW/m°C)
K	Constant for Bankoff model
l	Neutron generation time(s)
M	Molecular weight
n	Neutron density(1/m^3)
Nu	Nusselt number
N_{pch}	Phase change nunber
N_{sub}	Subcooling number
p_r	Reduced pressure
P	Pressure(MPa)
Pr	Prandtl number
q	Heat flux(kJ/m^2)
Re	Reynolds number
S	Suppression factor
t	Time(s)
T	Temperature(°C)
u	Velocity(m/s)
x	quality

Greek symbols

α	Void fraction
β	Delyed neutrons fraction
Δt	Time step(s)
κ	Reactivity feedback coefficient
λ	Delyed neutrons decay constant /friction factor
ρ	Density(kg/m^3)/reactivity
ϕ	Two-phase pressure drop multiplier

Suscripts

e	exit
f	Fluid/liquid
g	gas
i	inner
in	inlet
o	outer
w	wall

Acknowledgement

The authors wish to thank the LLOYD'S REGISTER FOUNDATION for financial support. The LLOYD'S REGISTER FOUNDATION is an independent charity working to achieve advances in transportation, science, engineering and technology education, training and research worldwide for the benefit of all.

References

[1] GUO, Y., SU, G.H., and WANG, J.Q., *et al.*: Two-phase instability analysis in natural circulation loops of China advanced research reactor. Annals of Nuclear Energy, 2005. 32(4): p. 379-397.

[2] JAIN, P.K., and RIZWAN, U.: Numerical analysis of supercritical flow instabilities in a natural circulation loop. Nuclear Engineering and Design, 2008. 238(8): p. 1947-1957.

[3] HAJIZADEH, A., KAZEMINEJAD, H., and TALEBI, S.: Formulation of a fully implicit numerical scheme for simulation of two-phase flow in a vertical channel using the Drift-Flux Model. Progress in Nuclear Energy, 2018. 103: p. 91-105.

[4] ZOU, L., ZHAO, H., and ZHANG, H.: Application of high-order numerical schemes and Newton-Krylov method to two-phase drift-flux model. Progress in Nuclear Energy, 2017. 100: p. 427-438.

[5] HAJIZADEH, A., KAZEMINEJAD, H., and TALEBI, S.: An advanced new fully implicit numerical method for two-phase flow subchannel analysis based on the Drift Flux Model. Annals of Nuclear Energy, 2017. 108: p. 351-365.

[6] ZOU, L., ZHAO, H., and ZHANG, H.: Numerical implementation, verification and validation of two-phase flow four-equation drift flux model with Jacobian-free Newton–Krylov method. Annals of Nuclear Energy, 2016. 87: p. 707-719.

[7] HIBIKI, T., and ISHII, M.: One-dimensional drift-flux model and constitutive equations for relative motion between phases in various two-phase flow regimes. International Journal of Heat and Mass Transfer, 2003. 46(25): p. 4935-4948.

[8] NAYAK, A.K., DUBEY, P., and CHAVAN, D.N., *et al.*: Study on the stability behaviour of two-phase natural circulation systems using a four-equation drift flux model. Nuclear Engineering and Design, 2007. 237(4): p. 386-398.

[9] PAUL, S., and SINGH, S.: A density variant drift flux model for density wave oscillations. International Journal of Heat and Mass Transfer, 2014. 69: p. 151-163.

[10] CHUNG, Y.J., JUN, I.S., and KIM, S.H., *et al.*: Development and assessment of system analysis code, TASS/SMR for integral reactor, SMART. Nuclear Engineering and Design, 2012. 244: p. 52-60.

[11] CHUNG, Y.J., KIM, S.H, and LEE, G.H., *et al.*: Applicability of TASS/SMR using drift flux model for SMART LOCA analysis. Nuclear Engineering and Design, 2013. 262: p. 228-234.

[12] CHUNG, Y.J., KIM, S.H., and LIM, S.W., *et al.*: TASS/SMR code improvement for small break LOCA applicability at an integral type reactor, SMART. Nuclear Engineering and Design, 2015. 295: p. 221-227.

[13] Lee, Y.G., and PARK, G.C.: TAPINS: a thermal-hydraulic system code for transient analysis of a fully-passive integral PWR. Nuclear Engineering and Technology, 2013. 45(4): p. 439-458.

[14] RAO, Y.F., FUKUDA, K., and KANESHIMA, R.: Analytical study of coupled neutronic and thermodynamic instabilities in a boiling channel. Nuclear Engineering and Design, 1995. 154(2): p. 133-144.

[15] FURUYA, M., INADA, F., and VAN DER HAGEN, T.H.J.J.: Development of SIRIUS-N facility with simulated void-reactivity feedback to investigate regional and core-wide stability of natural circulation BWRs. Nuclear Engineering and Design, 2005. 235(15): p. 1635-1649.

[16] SHI, S., WU, Z., and LIU, Z., *et al.*: Experimental study of natural circulation instability with void reactivity feedback during startup transients for a BWR-type SMR. Progress in Nuclear Energy, 2015. 83: p. 73-81.

[17] T'JOEN, C., and ROHDE, M.: Experimental study of the coupled thermo-hydraulic–neutronic stability of a natural circulation HPLWR. Nuclear Engineering and Design, 2012. 242: p. 221-232.

[18] LEE, J.D., PAN, C., and CHEN, S.W.: Nonlinear dynamic analysis of a two-phase natural circulation loop with multiple nuclear-coupled boiling channels. Annals of Nuclear Energy, 2015. 80: p. 77-94.

[19] WANG, J., SUN, Y., and WANG, J., *et al.*: Numerical study of nuclear coupled two-phase flow instability in natural circulation system under low pressure and low quality. Nuclear Engineering and Design, 2011. 241(12): p. 4972-4977.

[20] DURGA PRASAD, G.V., and PANDEY, M.: Parametric effects on reactivity instabilities and nonlinear oscillations in a nuclear-coupled double channel natural circulation boiling system. Nuclear Engineering and Design, 2010. 240(5): p. 1097-1110.

[21] DITTUS, F.W., and BOELTER, L.M.K.: Heat transfer in automobile radiators of the tubular type. International Communications in Heat and Mass Transfer, 1985. 12(1): p. 3-22.

[22] LIU, Z., and WINTERTON, R.H.S.: A general correlation for saturated and subcooled flow boiling in tubes and annuli, based on a nucleate pool boiling equation. International Journal of Heat and Mass Transfer, 1991. 34(11): p. 2759-2766.

[23] PIERRE, C.C.S., and S.G. BANKOFF.: Vapor volume profiles in developing two-phase flow. International Journal of Heat and Mass Transfer, 1967. 10(2): p. 237-249.