Regression model for crack severity estimation in NPP

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Abstract: In-containment pipe failures in a nuclear power plant are being detected by measuring the humidity in the containment. However, incipient leaks and cracks are difficult to detect because traditional leak monitors are not sensitive to small leak rate changes and cannot be used for low-level leak rates and are limited to post-accident analysis of significant releases. In this work, we present an optimized data-driven Support Vector Regression (SVR) model. The proposed method can be integrated with the existing leak detector to form a robust hybrid diagnostic system, effective for detecting both incipient and large leakage in nuclear plant pipes. The SVR model estimates the size and location of incipient breaks using fault signatures, and the size estimation efficiency is evaluated using the mean squared error values (MSE). To obtain efficient predictive model and minimize false alarm rate, Genetic Algorithm is utilized for feature selection purposes. To demonstrate the method and evaluate the generalization capability of the predictive model, cracks of various severities at the inlet plenum of CNP300 NPP is simulated with RELAP5/SCDAP Mod4.0 code. The SVR's relative error (MSE) with and without feature selection algorithms were compared using different solver algorithms. The result shows better performance for the model built with features selected by GA. The model also diagnose fault faster than conventional techniques.

Keyword: fault diagnosis; support vector regression; feature selection algorithm

1. Introduction

NPP online condition monitoring has developed from routine noise analysis technique for plant parameters to beyond sensor condition monitoring such as reactor internal vibration, leak detection, as well as performance evaluation of rotating parts and valves. Moreover, the presence of huge database provides better insights into the current state of the plant. Recently, the database has been indispensable in the utilization of the machine learning models and soft computing approaches for timely fault diagnosis. A few applications of majorly signal-based Fault detection and isolation (FDI) techniques for instrument calibration monitoring, instrument dynamic performance monitoring, equipment condition monitoring, reactor core monitoring, loose part monitoring, and some transients are reviewed in [1]

Some challenges affecting the practical FDI implementation in operating plants are the complexity of process dynamics, limited ranges of validity of the models, incomplete uncertain data, and model complexity ^[2]. To address these issues, a number of

researchers ^{[3][4]} have proposed the development and utilization of distributed Support Vector Regression (SVR) algorithms with various capabilities for FDI. Ye et al. [4] present a wavelet and SVR based method for locating grounded faults in radial distribution systems. The method utilizes traveling wave data recorded at a substation and used the maxima of modal components in each scale as the candidate features for training an SVR. A comprehensive method for integrating the predictive capability of two different intelligent systems to a knowledge-based operator support system for nuclear plant fault diagnosis is also presented in ^[5], although the architectures of the support vector regression can be more easily determined than that of neural networks. Ding and Fang^[6] utilized particle filter and nonlinear regression to predict faults in a nonlinear stochastic system with incipient faults. The effectiveness of the proposed method is verified by the simulations of the three-tank system. Liu et al.^[7], proposed the hybrid of Elman Neural Network (ENN) and Signed Directed Graph (SDG) for fault recognition. In [8], wavelet-SVM model was used to detect broken rotor faults in induction machine, and PCA was also introduced for feature extraction.

A practical limitation of SVR is in the rate of false alarm generation, as a result of redundant features, signal noises and model uncertainties. NPP operations characterized disturbances, are by noisv measurements, and state variations. A number of false alarms are generated as a result of these transients. Also, uncertainties such as model abstractions, as well as high background noise as found in NPPs can obscure fault detection by raising false alarms. Moreover, regression models that involve a large number of redundant features may result in reduced learning speed, performance degradation, and increased probability of over-fitting [9]. To compensate for the model uncertainties as a result of plant fluctuations and instability, and to avoid over-parameterization of the SVR model, there is a need for effective feature selection and extraction.

A number of feature selection algorithm and non-evolutionary statistical metrics such as Principal Component Analysis, Greedy Search, Sequential Feature Selection, and Bayesian Optimization have been explored to assess features' discriminative power ^[8, 10]. However, some of these methods suffer from local optima, high computational cost, and generalization problems. Comparatively, swarm intelligence algorithm and evolutionary computation such as Genetic Algorithms (GA), Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), among others have been shown to possess great capabilities in finding global optima. Specifically, GA has been shown to demonstrate consistent superiority over other methods in undertaking feature selection problems by employing an evolutionary and swarm-based strategy to yield multiple solutions for complex and non-linear problems ^[11].

Consequently, we propose an optimized support vector regression model to diagnose incipient, low-level fault and specifies the severity of the fault. We acknowledge that selecting features that are signatures of specific faults would improve the generalization ability of the model and reduce false alarm rate. Hence, to select appropriate features to train the SVR model, we experiment with GA feature selection algorithm. To test the method presented in this study, incipient cracks in the inlet plenum of a steam generator event is simulated using a thermal-hydraulic system code, RELAP5/Mod4.0.

Section 2 presents the utilization of the proposed method for fault detection. Fault modeling and simulation technique to obtain plant fault signatures event are presented in Section 3, and the SVR system development and evaluation results are presented in section 4. In Section 5, we summarized our findings and state the future direction.

2. Severity estimation with SVR model

2.1 Theory of SVR

Support Vector Regression (SVR) is a kernel-based machine learning approach, applied to tasks such as function approximation and regressive parameter estimation. SVR has been successfully used to handle regression problems and has been showed to achieve a good result when applied to forecast nonlinear systems ^[12]. The main feature of the algorithm is the use of a nonlinear kernel transformation to map the input variables into a feature space such that the relation with the output variable becomes linear in the transformed space. In this work, support vector regression algorithms output a real-valued response to set of non-linear predictors based on the non-linear kernel transformation function. a function that transforms the input into induced high dimensional feature space suitable for linearizing non-linear problems.

Compared to normal regression, SVR function does not change with additional samples, as long as the deviation introduced by the sample is less than a threshold $\mathcal{E}^{[13]}$. For samples with threshold beyond \mathcal{E} , there exists a \mathcal{E} -insensitive loss function C that is used to penalize such errors. The loss function gives a sparse representation of the decision rule, ensures the existence of the global minimum and optimizes the generalization bound, giving the algorithm a significant representational advantage. In SVR modeling, we consider an input $x_i \in \Re^D$; mapped into a z-dimensional feature space, by a non-linear kernel function, where a linear model f(x, w) is constructed in the feature space such that:

$$f(x) = \sum_{j=1}^{z} \omega_j \phi_j(x) + b \tag{1}$$

Where $\phi_j(x)$, j = 1....z is the set of non-linear transformation, ω is the weighing parameters, and b specifies the location of the hyper-plane bias away from the origin. For ε -insensitive SVR, the quality of estimation is measured by the loss function and the goal is to find the weight ω and bias b that minimizes the loss function given by:

$$L_{\varepsilon}(y, f(x, w)) = \begin{cases} 0 & \text{if } |y - f(x, w)| \le \varepsilon \\ |y - f(x, w)| = \varepsilon & \text{otherwise} \end{cases}$$
(2)

The SVR generalization capability varies for different tasks. For most non-linear regression task, nonlinear kernel technique is utilized to find the best separating boundary and the accurate response value for training instances that are not linearly separable. This kernel function maps the input space from lower dimension to higher dimensional input space, where linear separation is possible. The decision function learned by this approach simultaneously computes support vector regression models in place of multiple binary classifiers. Further discussion on SVR can be found in [13].

2.2 Feature selection using GA

Genetic algorithm (GA) is an evolutionary algorithm that simulates the process of natural selection, used for constrained and unconstrained solving both optimization problems. In GA, candidate solutions of the problem are encoded as a population of chromosomes. The algorithm repeatedly modifies a population of individual solutions. At each step, the algorithm randomly selects individuals from the current population and uses them as parents to produce the children for the next generation. GA incorporate randomness into their search procedure to escape local minima, and over successive generations, the population evolves toward an optimal solution ^[14]. In GA, a new population is formed using specific genetic operators such as crossover, reproduction, and mutation ^[15]. A standard representation of each chromosome is as a fixed-length array of bits. GA generates an initial population of feasible solutions and recombines them in a way to guide their search toward more promising areas of the search space. Each of these feasible solutions is encoded as a chromosome, also referred to as genotype, and each of these chromosomes will get a measure of fitness through a fitness function (evaluation or objective function), where low fitness value shows the better solution for minimization problems such as feature selection. GA has five main components: a random number generator, a fitness evaluation unit, a reproduction process, a crossover process, and a mutation operation. Reproduction selects the fittest candidates of the population, while crossover is the procedure of combining the fittest chromosomes and passing superior genes to the next generation, and mutation alters some of the genes in a chromosome [14].

Table 1:	Generalized	structure	of	Genetic Algorithm

GA():
Init_Rand_ P();
Evaluate_Fitness_P(V);
While (termination Criteria) do:
Parent selection;
Crossover with probability Pc to form a new offspring;
Mutation with probability Pm;
Fitness Calculation;
Survivor selection
If best solution found then:
Exit
Return the best solution

For this work, we utilized the GA feature selection algorithm developed by Ludwig ^[16] in MATLAB. Figure 1 shows the framework and expanded flow diagram of the SVR model training for the fault diagnostic system. Our focus is to use appropriate data filtering procedures based on the features selected by GA to design and train the SVR model, so as to eliminate redundant data, reduce false alarm events high-quality estimation. and obtain The pre-processing procedure for input data is also included to obtain more stable input and achieve higher estimation accuracy.



Fig.1 Framework and expanded flow diagram of the SVR model training.

3 Demonstration of the proposed fault diagnosis scheme

3.1 Fault modeling with RELAP5/Mod4.0

To test the proposed FDI method, we utilized the data derived from modeling and simulation of incipient cracks in CNP300 NPP hot leg inlet plenum. Unmitigated cracks could lead to large, guillotine break and loss of coolant accident. To obtain quality data that is representative of an operating NPP, RELAP5/SCDAP Mod4.0 thermal hydraulics code was used to model and simulate faults in CNP300 I NPP. Qinshan phase I NPP (CNP 300) is a Chinese owned two-loop 300MW (electric) PWR, located in Zhejiang Province, China. The RELAP5 code is a versatile and robust code based on a one-dimensional two-fluid model for two-phase flow. Figure 2 shows the nodalization diagram of a section of the reactor coolant system. For the simulation of SGTR fault in the RCS, a loop of the RCS is modeled. First, the full RCS primary loop is modeled, and the parameters are compared with the actual plant parameters, to confirm model accuracy. Table 2 shows the comparison of a few selected initial condition (steady state) parameters used as the actual operating parameters.

Sub-unit	Parame	Real	Simulated	Errors
	ters	values	Values	(%)
Steam	Feedwat	259.86	259.92	0.02
Generator	er flow	kg/s	kg/s	
	Steam outlet temperat ure	270.2°C	271.9°C	0.6
	Steam Pressure	5.5Mpa	5.52Mpa	0.36
	SG water level	10.47m	10.44m	0.21
Pressurizer	Pressuri zer pressure	15.4MPa	15.3MPa	0.60
	Pressuri zer level	5.400m	5.42m	0.37

 Table 2: Comparison of plant steady-state parameters

In modeling a system that reflects the status of RCS, the process we implemented is summarized in the following steps:

1. The full RCS is first simulated using the RELAP5 code. To confirm model accuracy, this full RCS model is then debugged and the model calculations are compared with the measured steady-state operating condition.

2. One of the loops in the RCS two-loop is selected to investigate the effectiveness of the method. The simulation model is debugged accordingly to ensure that simulated parameters are consistent with design parameters under all running conditions. The nodalization diagram of the RCS loop #1 modeling is as shown in Fig.2.

3. Incipient cracks are simulated in the inlet plenum of the steam generator of the loop #1. Figure 3 shows the implementation in RELAP5. The break simulating valve junctions provide a break flow path and the break size is determined by adjusting the flow area of the valve junction.

4. Data from the simulated faults are pre-processed and used to train the distributed SVR model.

5. To improve the performance of the SVR model, first different solver algorithm was tested on the model. Then an optimization algorithm is used to select features for the SVR.

6. The trained SVR is initialized after obtaining the related, real-time parameters indicating the steady state and the simulated fault states of the sub-unit, and the SVR performance is evaluated on the test data.

3.2. Model uncertainties and incipient faults description

This section describes the simulated fault sizes in the plant model. The in-built fault diagnosis system in most nuclear power plants can detect cracks around 70mm in length, with break flow rate above 0.5kg/s^[17]. Hence, we classify breaks below this threshold as incipient. Consequently, cracks in the inlet plenum without Safety Injection System (SIS) activation are selected as a case study to verify the SVR model. Since the break flow is within the makeup capacity of the charging system, an automatic reactor trip will not occur and if the faults are rapidly detected and diagnosed, controlled shutdown of the reactor would be performed utilizing the appropriate non-emergency procedures.

Figures 3, 4, 5, 6 and 7 are used to describe the cracks and the corresponding deviations in plant parameters as the size of the fault increases. The model nodalization for the crack in the inlet plenum is as depicted in Fig.3. After operating the plant in steady-state for 50s, the crack sizes are adjusted, and their corresponding parameters were observed for 1000s. Fig.4 shows the change in the break flow with time. Also, Fig.5 shows the changes in the primary temperature as the severity of the fault increases. Similarly, Fig.6 and 7 show the variation of the system pressure and pressurizer level as a function of time of fault occurrence.



Fig.2 Re-nodalized loop #1 of the Reactor Coolant System.



Fig.3 The model nodalization for the inlet plenum crack.



Fig.4 Break flow variation with different crack sizes.



Fig.5 Changes in primary temperature with crack sizes.



Fig.6 Deviations in system pressure during the event.



Fig.7 Pressurizer water level change with sizes.

4 Simulation result and analysis

For support vector regression task, the performance of the model relies heavily on the amount and quality of training samples as well as the selected kernel function. Also, selecting appropriate SVR hyper-parameter such as the kernel function and kernel width (for Gaussian kernel) and ε -insensitive loss requires diligent experimentation. To this end, we utilized the automatic hyper-parameter selection capability of MATLAB machine learning toolbox and experimented with different solver algorithms and kernel functions.

The steady state and fault condition plant data generated from RELAP5 was utilized for the selection of appropriate regression model. With 12-dimensional attributes (parameters) of 8510 observations (instances) as the data sample, a nonlinear SVR model was trained, and we experimented with different model parameters. First, to reduce convergence time for all experimental iterations, the data was standardized, and duplicate observations in the data were replaced with a single observation with weight equal to the sum of the weights of the corresponding removed duplicates. Subsequently, we partitioned the data into a training set (70%) and a test set (30%). The training set is used to fit the SVR model, and the test set is reserved to evaluate the performance of SVR. We utilized the MATLAB function fitrsvm, a function that trains and cross-validates a support vector regression model on а low-through moderate-dimensional predictor data set. The fitrsvm function supports mapping the predictor data using kernel functions and supports three solver algorithm options - Sequential Minimal Optimization

(SMO), Iterative Single Data Algorithm (ISDA), and soft-margin minimization L1 via quadratic programming (L1QP) - for objective function minimization. Detail description of the solver algorithms can be found in ^[18]. Consequently, we experimented with different supported kernel functions with different solver algorithms and generated their respective relative errors (MSE). Table 3 shows the result obtained when SVR model was trained directly with 12 features. The table shows Re-substitution Mean Squared Error (average over 10 runs) without feature selection methods considered over Inlet plenum crack datasets for SVR model using different solver algorithm and kernel functions. The best result is shown in bold.

Table 3:	R-MSE	without	feature	selection	considered	over
	Ы	nlet plen	um crac	ek dataset		

No of Iteration	Solver	Kernel Function	Resub. MSE
608	SMO	Linear	0.0083
107676	ISDA	Linear	0.0204
9	L1QP	Linear	0.0084
1475	SMO	Gaussian	0.0244
289	ISDA	Gaussian	0.0633
7	L1QP	Gaussian	0.0702
1000000	SMO	Polynomial	6.4782e+07
1000000	ISDA	Polynomial	0.1565
7	L1QP	Polynomial	1.4770e+05

It is observed that the SVR model with soft-margin minimization via quadratic programming solver algorithm with Linear kernel function shows the minimum mean squared error of 0.0084 after 9 iterations. Also, it is observed that SMO with Linear kernel has the least error (0.0083) but it takes a longer time to converge compared with L1QP.

To further optimize the performance of the SVR model, all experiments were repeated using the features selected by GA. The optimization is an experimental search over four (4) features selected by GA. We ranked the features based on their fitness values, using the feature selection algorithms, and the MSE results obtained using different solver algorithms were compared. The required output is the regression with the minimum estimated

cross-validation loss (MSE) and the result is as shown below. With the GA feature selection algorithm, Fig.8 shows the fitness function plot for 8 generations, and Table 4 shows the MSE result. The table shows the Re-substitution Mean Squared Error (average over 10 runs,) of GA feature selection methods considered over inlet plenum crack datasets for SVR model using different solver algorithm and kernel functions. The best result for each number of features is shown in bold.

No of	Solver	Kernel	Resubstitutio
Iteration		Function	n MSE
4931	SMO	Linear	0.0080
205847	ISDA	Linear	0.0213
10	L1QP	Linear	0.0067
197	SMO	Gaussian	0.1218
179	ISDA	Gaussian	0.1395
8	L1QP	Gaussian	0.1266
1000000	SMO	Polynomial	1.2323e+11
1000000	ISDA	Polynomial	0.2244
8	L1QP	Polynomial	163.8688

It is observed in Table 4 that L1QP with Linear kernel have the least MSE value of 0.0067 after 10 iterations. A similar trend is noticed in Table 3, where Linear L1QP has less MSE value and the least number of iteration. However, the SVR model trained with GA selected features gives the least MSE. It is also seen that the proposed feature selection method reduces the false alarm rate by 20%. That is, there is 20% reduction in the false alarm rate, as the MSE value is reduced from 0.0084 to 0.0067.



Fig.8 Fitness function curve for GA.

Hence, comparing the algorithms' MSE values and their number of iterations in Tables 3 and 4, it is observed that GA with soft-margin minimization via quadratic programming (L1QP) solver algorithm and Linear kernel function gives the best fault prediction, and its MSE is selected as the threshold, δi , (maximum allowable MSE as in Fig.1) for the system. Another observation is that the least MSE value for the estimated parameters will form the basis of the fault detection threshold which will be selected later as the criterion for the occurrence of a fault by the fault diagnosis process.

In summary, on the representative data derived from the simulation of cracks in the inlet plenum of the CNP 300 (Qinshan I) NPP steam generator, the best predictive SVR model is obtained using L1QP solver algorithm and Linear kernel function, with GA algorithm applied to select the most informative features.

5. Conclusion and future work

This work presents soft computing technique for crack detection in NPP, implementable on the plant's operator support system. We propose a system where support vector regression model detects incipient, low-level faults and specifies the severity of the fault. The purpose of the SVR is to estimate the size of fault and to trigger an alarm in order to achieve timely intervention of the operator. To optimize the design of the SVR model, we utilized features that are signatures of the faults, selected using Genetic Algorithm. The data for training the model is obtained from the simulation of cracks in the Chinese CNP300 pressurized water reactor, using RELAP5/SCDAP thermal-hydraulic system code. First, a nonlinear SVR model was trained with 12-dimensional parameters of 8510 observations as the training sample, and relative errors (MSE) were compared using different SVR solver algorithms kernel functions. Then, GA feature selection algorithm is applied to select important features for the training of the SVR model. Different results from the feature selection algorithm were compared, and the best result, indicating the least MSE, is selected as the optimum design for the SVR. The feature selection performance curves for GA is shown in Fig.8 and the resulting performance evaluation of the SVR model is described in Table 4.

The diagnostic result derived from the experiment shows that:

- 1. SVR model designed with soft-margin minimization via quadratic programming solver algorithm and linear kernel function has the best performance for the simulated fault.
- 2. The proposed method diagnoses incipient crack event faster than conventional methods for the leak rate investigated.
- The method can be combined with existing leak detection techniques to form a robust and efficient diagnostic method implementable on operator support system.

In practice the choice of which feature selection method to use for a particular system calls for procedures of comparison and validation in order to guide the choice of the adequate approach for a given situation. Our next research focus would be on the comparative analysis of feature selection algorithms and procedure to adaptively train the SVR model online.

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